## BUSINESS FINANCE

The Primary textbook for the course is:

- Essentials of Corporate Finance, by Ross, Westerfield and Jordan, fourth edition, McGraw Hill Publishers, ISBN 0-07-121057-7

Reference books will be:

- Introduction to Finance by Lawrence J. Gitman and Jeff Madura, Addison-Wesley Publishers
- Foundations of Financial Management by Stanley B. Block and Geoffrey A. Hirt, McGraw Hill Publishers


## Course Contents

- An overview of Financial Environment
- Financial Statements, Taxes and Cash Flows
- Time Value of Money and Discounted Cash Flow Valuation
- Valuation of Stocks and Bonds
- Net Present Value and other Investment Criteria
- Capital Investment Decision
- Risk and Return
- Cost of Capital
- Leverage and Capital Structure
- Raising Capital
- Working Capital Management
- Dividends


## Finance: A Quick Look

- Four Basic Areas
- Business Finance
- Investments
- Financial Institutions
- International Finance


## Business Finance

Addresses the following three questions:

- What long-term investments should the firm engage in?
- How can the firm raise the money for the required investments?
- How much short-term cash flow does a company need to pay its bills?


## Investments

- Deals with financial assets such as stocks and bonds.
- It covers the following issues:
- Pricing Financial Assets
- Associated Risks and Rewards
- Determining best mixture of financial investment
- Career opportunities in investment
- Stock Brokerage
- Portfolio Management
- Security Analysis


## Financial Institutions

- Businesses dealing in financial matters
- Banks and Insurance companies


## International Finance

- Covers international aspects of corporate finance, investment and financial institutions.


## WHY STUDY FINANCE?

## Why Study Finance?

- Marketing and Finance
- Marketers have to work with budgets
- Need to get greatest payoffs from marketing expenditures and programs
- Cost and Benefit analysis of projects
- So, finance is vital for:
- Marketing research
- Design of marketing and distributions channels
- Product pricing
- Accounting and Finance
- Accountants are required to make financial decisions as well as understand the implications of new financial contracts
- Financial analysts make extensive use of accounting information
- Management and Finance
- Business Strategy is always disastrous if financial planning is not adhered to.

What is Business Finance?

- In order to start any new business, the following issues become vital
- What long-term investment should be taken on?
- From where to get the long-term financing to pay for investment? Bring in other owners or borrow the money?
- How to manage everyday financial activities?

The Financial Manager
To create value, the financial manager should:

- Try to make smart investment decisions.
- Try to make smart financing decisions.

Hypothetical Organization Chart


Business Finance and Financial Manager

- Financial Management Decisions
- Capital Budgeting
- Capital Structure
- Working Capital Management

Financial Management Decisions

- Capital Budgeting
- The process of planning and managing a firm's long-term investments
- Financial managers concern with how much, when and how likely is cash expected to receive
- Evaluating the size, timing and risk of future cash flows is the essence of capital budgeting

| Total Value of Assets <br> Current <br> Assets <br> Fixed Assets <br> - Tangible <br> - Intangible | Total Firm Value to Investors <br> Current Liabilities |
| :---: | :---: |
|  | Shareholders' <br> Equity |

## Capital Budgeting Decision



The Capital Structure Decision


Fixed Assets

- Tangible
- Intangible


## Current Liabilities

How can the firm raise the money for the required investments?

Shareholders'
Equity

Capital Structure

- The value of the firm can be thought of as a pie.
- The goal of the manager is to increase the size of the pie.
- The Capital Structure decision can be viewed as how best to slice up the pie.
- If how you slice the pie affects the size of the pie, then the capital structure decision matters.


The Net Working Capital Investment Decision


The Corporate Firm

- The corporate form of business is the standard method for solving the problems encountered in raising large amounts of cash.
- However, businesses can take other forms.


## Forms of Business Organization

- Three major forms
- Sole proprietorship
- Partnership
- General
- Limited
- Corporation
- Limited Liability Company


## Sole Proprietorship

- Advantages
- Easiest to start
- Least regulated
- Single owner keeps all the profits
- Taxed once as personal income
- Disadvantages
- Limited to life of owner
- Equity capital limited to owner's personal wealth
- Unlimited liability
- Difficult to sell ownership interest


## Partnership

- Two or more owners (partners)
- General partnership: all partners share in gains and losses and all have unlimited liability for all partnership debts
- Limited partnership: one or more general partners will run the business and have unlimited liability but there will be one or more limited partners who do not actively participate in the business and their liability is limited to their contribution.
- Advantages
- Two or more owners
- More capital available
- Relatively easy to start
- Income taxed once as personal income
- Disadvantages
- Unlimited liability
- General partnership
- Limited partnership
- Partnership dissolves when one partner dies or wishes to sell
- Difficult to transfer ownership


## THE CORPORATE FIRM

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## Corporation

- A business created as a distinct legal entity owned by one or more individuals or entities.
- Forming of corporation involves preparing
- Charter including corporation's name, intended life, business purpose and number of shares
- Set of bylaws which describes the regulations for the business


## Separation of Ownership and Control



## Corporation

- Advantages
- Limited liability
- Unlimited life
- Separation of ownership and management
- Transfer of ownership is easy
- Easier to raise capital
- Disadvantages
- Separation of ownership and management
- Double taxation (income taxed at the corporate rate and then dividends taxed at personal rate)


## Goal of the Corporate Firm

- The traditional answer is that the managers of the corporation are obliged to make efforts to maximize shareholder wealth.
- Alternatively, the goal of the financial manager is to maximize the current value per share of the existing stock.


## The Set-of-Contracts Perspective

- The firm can be viewed as a set of contracts.
- One of these contracts is between shareholders and managers.
- The managers will usually act in the shareholders' interests.
- The shareholders can devise contracts that align the incentives of the managers with the goals of the shareholders.
- The shareholders can monitor the managers' behavior.
- This contracting and monitoring is costly.


## The Agency Problem

- Agency relationship
- Principal hires an agent to represent their interest
- Stockholders (principals) hire managers (agents) to run the company
- Agency problem
- Conflict of interest between principal and agent
- Management goals and agency costs


## Managerial Goals

- Managerial goals may be different from shareholder goals
- Expensive perquisites
- Survival
- Independence
- Increased growth and size are not necessarily the same thing as increased shareholder wealth.


## Do Shareholders Control Managerial Behavior?

- Shareholders vote for the board of directors, who in turn hire the management team.
- Contracts can be carefully constructed to be incentive compatible.
- There is a market for managerial talent-this may provide market discipline to the managersthey can be replaced.
- If the managers fail to maximize share price, they may be replaced in a hostile takeover.


## Managing Managers

- Managerial compensation
- Incentives can be used to align management and stockholder interests
- The incentives need to be structured carefully to make sure that they achieve their goal
- Corporate control
- The threat of a takeover may result in better management
- Other stakeholders

THE FIRM AND THE FINANCIAL MARKETS


Financial Markets

- Primary Market
- When a corporation issues securities, cash flows from investors to the firm.
- Usually an underwriter is involved
- Secondary Markets
- Involve the sale of "used" securities from one investor to another
- Securities may be exchange traded or trade over-the-counter in a dealer market.

Financial Markets


## Dealer vs. Auction Markets

- Auction markets are different from dealer markets in two ways:
- Trading in a given auction exchange takes place at a single site on the floor of the exchange.
- Transaction prices of shares are communicated almost immediately to the public.
- Listing

The Balance Sheet

- An accountant's snapshot of the firm's accounting value as of a particular date.
- The Balance Sheet Identity is: Assets $\equiv$ Liabilities + Stockholder's Equity
- When analyzing a balance sheet, the financial manager should be aware of three concerns: Accounting Liquidity, Debt versus Equity, and Value versus Cost


## The Balance-Sheet Model of the Firm

## Total Value of Assets

## Current Assets

## Fixed Assets

1. Tangible
2. Intangible

Total Firm Value to Investors
Current Liabilities
Long-Term Debt

## Shareholders' Equity

## Net Working Capital

- Net Working Capital = Current Assets - Current Liabilities
- NWC > 0 when Current Assets > Current Liabilities
- NWC < 0 when Current Assets < Current Liabilities
- NWC $=0$ when Current Assets $=$ Current Liabilities
- NWC usually grows with the firm for the healthy firms.

The Balance-Sheet Model of the Firm
The Net Working Capital Investment Decision


## Building the Balance Sheet

- A firm has:
- Current Assets of $\$ 100$,
- Net Fixed Assets of $\$ 500$,
- Short-term Debt of $\$ 70$, and
- Long-term Debt of $\$ 200$
- Now...
- Total Assets are $\$ 100+500=\$ 600$
- Total Liabilities are $\$ 70+200=\$ 270$
- Shareholders' Equity is $\$ 600-270=\$ 330$

Building the Balance Sheet

| Assets | Liabilities and <br> Shareholders' Equity |  |  |
| :--- | :---: | :--- | :---: |
| Current Assets | $\$ 100$ | Current Liabilities | $\$ 70$ |
| Net Fixed Assets | 500 | Long Term Debt |  |
|  |  | Shareholders' equity | 200 |
| Total Assets | $\$ \mathbf{6 0 0}$ | Total liabilities and <br> Shareholders' equity | $\$ \mathbf{\$ 6 0 0}$ |

## The Balance Sheet of the XYZ Corporation

| XYZ CORPORATION <br> Balance Sheet <br> 20X2 and 20X1 <br> (in \$ millions) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\underline{\text { Assets }}$ | 20X2 | 20X1 | Liabilities (Debt) and Stockholder's Equity | 20X2 | 20X1 |
| Current assets |  |  | Current Liabilities |  |  |
| Cash and equivalents | \$140 | \$107 | Accounts payable | \$213 | \$197 |
| Accounts receivable | 294 | 270 | Notes payable | 50 | 53 |
| Inventories | 269 | 280 | Accrued expenses | 223 | 205 |
| Other | 58 | 50 |  |  |  |
| Total current assets | \$761 | \$707 | Total current liabilities | \$486 | \$455 |
| Fixed assets |  |  | Long-term liabilities |  |  |
| Property, plant, and equipment | \$1423 | \$1274 | Deferred taxes | \$117 | \$104 |
| Less accumulated depreciation | -550 | -460 | Long-term debt | 471 | 458 |
| Net property, plant, and equipment | \$873 | \$814 | Total long-term liabilities | \$588 | \$562 |
| Intangible assets and other | 245 | 221 | Stockholder's equity |  |  |
| Total fixed assets | \$1,118 | \$1,035 | Preferred stock | \$39 | \$39 |
|  |  |  | Common stock (\$1 per value) | 55 | 32 |
|  |  |  | Capital surplus | 347 | 327 |
|  |  |  | Accumulated retained earnings | 390 | 347 |
|  |  |  | Less treasury stock | -26 | -20 |
|  |  |  | Total equity | \$805 | \$725 |
| Total assets | \$1,879 | \$1,742 | Total liabilities and stockholder's equity | \$1,879 | \$1,742 |

## Notes:

1. Long-term debt rose by ( $\$ 471$ million- $\$ 458$ million) $\$ 13$ million. This is the difference between $\$ 86$ million new debt and $\$ 73$ million in retirement of old debt.
2. Treasuy stock rose by $\$ 6$ million. This reflects the repurchase of $\$ 6$ million stock.
3. XYZ Corporation reports $\$ 43$ million in new equity. The company issued 23 million shares at a price of $\$ 1.87$. The par value of common stock is increased by $\$ 23$ million and capital suptlus is increased by $\$ 20$ million.

## Balance Sheet Analysis

- When analyzing a balance sheet, the financial manager should be aware of three concerns:
- Accounting liquidity
- Debt versus equity
- Value versus cost


## Accounting Liquidity

- Refers to the ease and quickness with which assets can be converted to cash.
- Current assets are the most liquid.
- Some fixed assets are intangible.
- The more liquid a firm's assets, the less likely the firm is to experience problems meeting shortterm obligations.
- Liquid assets frequently have lower rates of return than fixed assets.


## The Balance Sheet of the XYZ Corporation



## Debt versus Equity

- Generally, when a firm borrows it gives the bondholders first claim on the firm's cash flow.
- Thus shareholder's equity is the residual difference between assets and liabilities.

Shareholders' Equity = Assets - Liabilities

- The Use of debt in a firm's capital structure is called "Financial Leverage"
- The more debt a firm has (as a percentage of assets) the greater is the degree of financial leverage
- Debt acts as a lever in the sense that it magnifies both gains and losses


## Value versus Cost

- The true value of any asset is its market value, which is simply the amount of cash we would get if we actually sold it.
- The values shown on the balance sheet for the firm's assets are book values and generally are not what the assets are actually worth.
- Under the Accounting standards audited financial statements of firms carry assets at historical cost.
- For current assets, market value and book value might be somewhat similar since they are bought and converted into cash over a relatively short span of time.
- For fixed assets, it's very unlikely that the actual market value of an asset is equal to its book value.
- Example: Land purchased for railroads a century ago
- Similarly the owner's equity figure on the balance sheet and the true market value of the equity need not be related.
- For Financial Managers, accounting value of the equity is not a matter of concern rather it is the market value of the shares that matters.


## Market vs. Book Value

- K Corporation has fixed assets with a book value of $\$ 700$ and an appraised market value of \$1,000
- Net working capital is $\$ 400$ on the books but approx. $\$ 600$ would be realized if the current accounts were liquidated
- K has $\$ 500$ in long-term debt, both book \& market value
- What is the book value of the equity?
- What is the market value?

| K Corporation <br> Balance Sheet <br> Market Value vs. Book Value |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Assets | Book | Market | Liabilities (Debt) and Stockholder's Equity | Book | Market |
| Net working Capital | \$400 | \$600 | Long-term debt | \$500 | \$500 |
| Net Fixed Assets | \$700 | \$1,000 | Shareholders' equity | \$600 | \$1,100 |
| TOTAL | \$1,100 | \$1,600 | TOTAL | \$1,100 | \$1,600 |

## THE INCOME STATEMENT

## The Income Statement

- If we think of the balance sheet as a snapshot then we can think of income statement as a video recording covering before and after the picture.
- The income statement measures performance over a specific period of time
- The accounting definition of income is:

$$
\text { Revenue - Expenses } \equiv \text { Income }
$$

## XYZ Corporation Income Statement

| The operations section |
| :--- |
| of the income statement |
| reports the firm's |
| revenues and expenses |
| from principal |
| operations. |



| XYZ CORPORATION <br> Income Statement <br> 20X2 <br> (in \$ millions) |  |  |
| :--- | ---: | ---: |
| Totaloperating revenues <br> Cost of goods sold <br> Selling, general, and administrative expenses <br> Depreciation <br> Operating income | $\$ 2,262$ |  |
| Other income | $-1,655$ |  |
| Earnings before interest and taxes | -327 |  |
| Interest expense | -90 |  |
| Pretax income | $\$ 190$ |  |
| Taxes | 29 |  |
| Current: \$71 | $\$ 219$ |  |
| Deferred: $\$ 13$ | -49 |  |
| Net income | $\$ 170$ |  |
| Retained earnings | -84 |  |
| Dividends |  |  |



| Netincome is the "bottom line". | Craxes <br> Current: $\$ 71$ <br> Deferred: $\$ 13$ |  |
| :--- | :--- | :--- |
|  | Net income |  |
|  | Retained earnings |  |
|  | Dividends |  |

## Income Statement Analysis

- There are three things to keep in mind when analyzing an income statement:
- Generally Accepted Accounting Principles (GAAP)
- Non-Cash Items
- Time and Costs


## Income Statement Analysis

- Generally Accepted Accounting Principles (GAAP)
- "The Realization principle" is to recognize revenue when the earning process is complete, i.e. revenue is recognized at the time of sale, which need not be the same as time of collection.
- "The matching principal" of GAAP dictates that revenues be matched with expenses. Thus, income is reported when it is earned, even though no cash flow may have occurred.


## Income Statement Analysis

- Non-Cash Items
- The primary reason that accounting income differs from cash flow is that income statement contain non-cash items
- Depreciation is the most apparent. No firm ever writes a check for "depreciation".
- The depreciation deduction is simply an application of the matching principle in accounting.
- Another noncash item is deferred taxes, which does not represent a cash flow.


## Income Statement Analysis

- Time and Costs
- In the short run, certain equipment, resources, and commitments of the firm are fixed, but the firm can vary such inputs as labor and raw materials.
- In the long run, all inputs of production (and hence costs) are variable.
- Financial accountants do not distinguish between variable costs and fixed costs. Instead, accounting costs usually fit into a classification that distinguishes product costs from period costs.


## Income Statement Analysis

- Time and Costs
- Product cost include such things as raw materials, direct labor and manufacturing overhead and are reported on the income statement as the cost of goods sold, but they include both fixed and variable costs.
- Period costs include selling, general, and administrative expenses which may be fixed as well as variable.


## Taxes

- One of the largest cash outflows that a Corporate firm experiences.
- The size of tax is determined through the tax schedule issued by the Central Board of Revenue.
- Taxes for partnerships and proprietorship are computed using the personal income tax schedules.

| Model Tax Rates |  |
| :--- | :---: |
| Taxable Income | Tax Rate |
| $\$ 0-50,000$ | $15 \%$ |
| $50,001-75,000$ | 25 |
| $75,001-100,000$ | 34 |
| $100,001-335,000$ | 39 |
| $335,001-10,000,000$ | 34 |
| $10,000,001-15,000,000$ | 35 |
| $15,000,001-18,333,333$ | 38 |
| $18,333,334+$ | 35 |

## Average vs. Marginal Tax Rates

- Average tax rate is tax bill divided by the taxable income or the percentage of the income that goes to pay taxes
- Marginal tax rate is the extra tax you would pay if you earn one more dollar.
- Suppose a Corporation has a taxable income of $\$ 200,000$. So the Tax calculation will be:

| $\$ 50,000 \times 15 \%$ | $\$ 7,500$ |
| :---: | ---: |
| $(\$ 75,000-50,000) \times 25 \%$ | 6,250 |
| $(\$ 100,000-75,000) \times 34 \%$ | 8,500 |
| $(\$ 200,000-100,000) \times 39 \%$ | 39,000 |
| TOTAL | $\$ \mathbf{6 1 , 2 5 0}$ |

- Our total tax is $\$ 61,250$
- Average tax rate is $\$ 61,250 / 200,000=30.625 \%$
- Marginal rate is $39 \%$


## Flat Tax rate

- There is only one tax rate and this rate is same for all income levels.
- With such a tax, the marginal tax rate is always same as the average tax rate.
- The model tax rate schedule presented earlier represents a modified flat-rate tax, which becomes a true flat rate for the highest incomes.
- Let's take another view


## Average vs. Marginal Tax Rates

| Taxable Income | Marginal Tax <br> Rate | Total Tax | Average tax Rate |
| :---: | :---: | :---: | :---: |
| $\$ 45,000$ | $15 \%$ | $\$ 6,750$ | $15.00 \%$ |
| 70,000 | 25 | 12,500 | 17.86 |
| 95,000 | 34 | 20,550 | 21.63 |
| 250,000 | 39 | 80,750 | 32.30 |
| $1,000,000$ | 34 | 340,000 | 34.00 |
| $17,500,000$ | 38 | $6,100,000$ | 34.86 |
| $50,000,000$ | 35 | $17,500,000$ | 35.00 |
| $100,000,000$ | 35 | $35,000,000$ | 35.00 |

## Average vs. Marginal Tax Rates

- We see that the more a corporation makes, the grater is the percentage of taxable income paid in taxes.
- So the average tax rate never goes down, even though the marginal rate does.
- It will normally be the marginal tax rate that is relevant for financial decision making, since any new cash flows will be taxed at the marginal rate


## Cost of a Tax Deductible Expense

- The businesspersons often say that a tax-deductible item, such as interest on loans, travel expenditures, or salaries, costs substantially less than the amount spent on after-tax basis.
- Let's examine two corporations - one pays $\$ 100,000$ in interest, while other has no interest expense.

|  | Corporation A | Corporation B |
| :--- | ---: | ---: |
| Earnings before interest and taxes | $\$ 400,000$ | $\$ 400,000$ |
| - Interest Expense | $\mathbf{1 0 0 , 0 0 0}$ | $\mathbf{0}$ |
| Earnings before taxes (taxable income) | 300,000 | 400,000 |
| - Taxes @35\% | 105,000 | 140,000 |
| Earnings after taxes | $\$ 195.000$ | $\$ 260,000$ |
| Difference in earnings after taxes | $\$ 65,000$ |  |

It can also be computed as: Interest Expense (1-Tax rate)

$$
\begin{aligned}
& =\$ 100,000(1-35 \%) \\
& =\$ 65,000
\end{aligned}
$$

## Cost of a Tax Deductible Expense

- Interest is deducted from earnings before determining taxable income, thus saving $\$ 35,000$ in taxes and costing only $\$ 65,000$ on a net basis.
- Because a dividend on common stock is non-tax-deductible, we say it cost $100 \%$ of amount paid. From a purely corporate cash flow point of view, the firm would be indifferent between paying $\$ 100,000$ in interest and $\$ 65,000$ in dividends.


## DEPRECATION AS A TAX SHIELD

- Although depreciation is not a new source of funds, it provides the important function of shielding part of our income from taxes.
- Again, we take the same two corporations - one charges off $\$ 100,000$ in depreciation, other charges off none.

|  | Corporation A | Corporation B |
| :--- | ---: | ---: |
| Earnings before interest and taxes | $\$ 400,000$ | $\$ 400,000$ |
| - Depreciation | $\mathbf{1 0 0 , 0 0 0}$ | $\mathbf{0}$ |
| Earnings before taxes (taxable income) | 300,000 | 400,000 |
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| Earnings after taxes | $\$ 195.000$ | $\$ 260.000$ |
| + Dep. charged without cash outlay | $\mathbf{1 0 0 , 0 0 0}$ | $\mathbf{0}$ |
| Cash flow | $\$ 295,000$ | $\$ 260,000$ |
| Difference | $\mathbf{\$ 3 5 , 0 0 0}$ |  |

It can also be computed as: Depreciation x Tax rate

$$
\begin{aligned}
& =\$ 100,000 \times 35 \% \\
& =\$ 35,000
\end{aligned}
$$

## Depreciation as a Tax Shield

- Corporation A enjoys $\$ 35,000$ more in cash flow, since depreciation shielded $\$ 100,000$ from taxation in Corporation A and saved $\$ 35,000$ in taxes, which eventually appeared in cash flows.


## Financial Cash Flow

- In finance, the most important item that can be extracted from financial statements is the actual cash flow of the firm.
- Since there is no magic in finance, it must be the case that the cash received from the firm's assets must equal the cash flows to the firm's creditors and stockholders.


## Cash Flow from Assets $=$ Cash Flow to Creditors + Cash Flow to Stockholders

## Cash Flow from Assets

- Cash flow from assets involves three components
- Operating Cash flow
- Capital Spending
- Change in Net Working capital


## Operating Cash Flow

- This refers to the cash flow that results from the firm's day-to-day activities of producing and selling
- Expenses related to firm's financing of its assets are not included since they are not operating expenses
- To calculate OCF, we calculate revenues minus costs (including taxes being paid in cash), but don't include:
- Depreciation since it is not a cash out flow
- Interest because it is a financing expense
- OCF is very significant as it tells whether or not the firm's cash inflows from its business operations are sufficient to cover its everyday cash outflows.
- So, a negative operating cash flow is a sign of trouble.


## XYZ CORPORATION

Financial Cash Flow 20X2
(in $\$$ millions)

| Cash Flow of the Firm |  |
| :--- | ---: |
| Operating cash flow | $\$ 238$ |
| Capital spending | $(173)$ |
| Additions to net working capital | $(23)$ |
| Total | $\$ 42$ |
| Cash Flow of Investors in the Firm |  |
| Debt | $\$ 36$ |
| Equity | 6 |
| Total | $\$ 42$ |


| Operating |  |
| :--- | :---: |
| Cash Flow |  |
| EBIT | $\$ 219$ |
| Depreciation | 90 |
| Current Taxes | $\underline{(71)}$ |
| OCF | $\underline{\mathbf{2 3 3}}$ |
|  |  |
|  |  |

## Capital Spending

- Some portion of the firm's cash flow is reinvested in the firm. Capital spending refers to the net spending on fixed assets (Purchases of fixed assets less sale of fixed assets).
- Net Capital Spending could be negative if the firm sold off more assets than it purchased.
- Depreciation of the respective assets is accounted for in this regard.

| XYZ CORPORATION <br> Financial Cash Flow 20X2 (in $\$$ millions) |  |
| :---: | :---: |
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## Capital Spending

Purchase of fixed assets $\$(198)$
Sales of fixed assets $\quad 25$
Capital Spending $\$(173)$
$\$ 173=\$ 149+\$ 24$ (increase in
property, plant, and equipment +
increase in intangible assets)

## Change in Net Working Capital

- It is the amount spent on Net Working Capital, and represents the net increase in current assets over current liabilities.


## XYZ CORPORATION

## Financial Cash Flow

20X2
(in \$ millions)

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| Equity | $\mathbf{6}$ |
| Total | $\mathbf{\$ 4 2}$ |

NWC grew to $\$ 275$ million in 20X2 from $\$ 252$ million in

20X1. This increase of $\$ 23$ million is the addition to NWC.

## XYZ CORPORATION <br> Financial Cash Flow 20X2 <br> (in \$ millions)

Cash Flow of the Firm
Operating cash flow \$238
Capital spending

Cash Flow of Investors in the Firm
Debt
Equity
\$36
Total

| 6 |
| ---: |
| $\$ 42$ |

Total Cash Flow of the Firm

## Cash Flow to Creditors

- Cash flow to creditors is calculated as interest paid less net new borrowing.


## XYZ CORPORATION <br> Financial Cash Flow 20X2 <br> (in \$ millions)

Cash Flow of the Firm

| Operating cash flow | $\$ 238$ |
| :--- | ---: |
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|  | $\$ 42$ |
| Cash Flow of Investors in the Firm |  |
| Debt | $\$ 36$ |
| Equity | 6 |
| Total | $\$ 42$ |


| Cash Flow to Creditors |  |
| :--- | ---: |
| Interest | $\$(49)$ |
| Retirement of debt | $\underline{(73)}$ |
| Debt service | $\$(122)$ |
| Proceeds from new debt sales | $\underline{86}$ |
| Total | $\underline{\$(36)}$ |

## Cash Flow to Stockholders

- Cash flow to stockholders is calculated as dividends paid less net new equity raised.

| XYZ CORPORATION |
| :---: |
| Financial Cash Flow |
| 20X2 |
| (in $\$$ millions) |

Cash Flow of the Firm
Operating cash flow \$238
Capital spending
Additions to net working capital
Total
Cash Flow of Investors in the Firm Debt
\$36

| Equity | 6 |
| :--- | ---: |
|  | $\$ 42$ |

apital spending (173)

## YZ CORPORATION 20X2 \$ millions)

## Cash Flow to Stockholders

Dividends \$(43)

Repurchase of Stock (6)
Cash to Stockholders \$(49)
Proceeds from new stock issue
Total
\$ (6)

| XYZ CORPORATION Financial Cash Flow 20X2 (in $\$$ millions) |  |
| :---: | :---: |
| Cash Flow of the Firm |  |
| Operating cash flow | \$238 |
| Capital spending | (173) |
| Additions to net working capital | (23) |
| Total | \$42 |
| Cash Flow of Investors in the Firm |  |
| Debt | \$36 |
| Equity | 6 |
| Total | \$42 |

## Total Cash Flow to Creditors and Stockholders.

## XYZ CORPORATION Financial Cash Flow 20X2 (in $\$$ millions)

| Cash Flow of the Firm |  |
| :--- | ---: |
| Operating cash flow | $\$ 238$ |
| Capital spending | $(173)$ |
| Additions to net working capital | $(23)$ |
| Total | $\$ 42$ |
| Cash Flow of Investors in the Firm |  |
| Debt | $\$ 36$ |
| Equity | 6 |
| Total | $\$ 42$ |

The cash from received from the firm's assets must equal the cash flows to the firm's creditors and stockholders:

CF (Assets)
$=$

CF (Creditors) + CF (Stockholders)

## Cash Flow Summary

Cash Flow identity:

- Cash flow from Assets $=$ Cash Flow to creditors + Cash flow to Stockholders
- Cash flow from Assets:

$$
\begin{aligned}
& \text { Cash flow from assets = } \text { Operating Cash Flow } \\
&- \text { Net Capital Spending } \\
& \quad-\text { Change in Net Working Capital } \\
& \text { Where, } \\
& \text { Operating cash flow = Earnings before Interest and taxes }+ \text { Depreciation }- \text { Taxes } \\
& \text { Net Capital Spending = Ending Net Fixed Assets - Beginning Net Fixed Assets + Depreciation } \\
& \text { Change in NWC = Ending NWC - Beginning NWC }
\end{aligned}
$$

- Cash flow to creditors (bondholders):

Cash flow to creditors $=$ Interest paid - Net new borrowings

- Cash flow to stockholders (owners):

Cash flow to stockholders $=$ Dividends Paid - Net new equity raised

- There is an official accounting statement called the statement of cash flows.
- This helps explain the change in accounting cash,
- The three components of the statement of cash flows are
- Cash flow from operating activities
- Cash flow from investing activities
- Cash flow from financing activities

The Balance Sheet of the XYZ Corporation

| XYZ CORPORATION <br> Balance Sheet <br> 20X2 and 20X1 <br> (in \$ millions) |  |  |
| :---: | :---: | :---: |
| Assets | 20X2 | 20X1 |
| Current assets: |  |  |
| Cash and equivalents | \$140 | \$107 |
| Accounts receivable | 294 | 270 |
| Inventories | 269 | 280 |
| Other | 58 | 50 |
| Total current assets | \$761 | \$707 |
| Fixed assets: |  |  |
| Property, plant, and equipment | \$1423 | \$1274 |
| Less accumulated depreciation | - 550 | - 460 |
| Net property, plant, and equipment | 873 | 814 |
| Intangible assets and other | 245 | 221 |
| Total fixed assets | \$1,118 | \$1,035 |
| Total assets | \$1,879 | \$1,742 |


| XYZ CORPORATION <br> Balance Sheet <br> 20X2 and 20X1 <br> (in \$ millions) |  |  |
| :---: | :---: | :---: |
| Liabilities (Debt) and Stockholder's Equity | 20X2 | 20X1 |
| Current Liabilities: |  |  |
| Accounts payable | \$213 | \$197 |
| Notes payable | 50 | 53 |
| Accrued expenses | 223 | 205 |
| Total current liabilities | \$486 | \$455 |
| Long-term liabilities: |  |  |
| Deferred taxes | \$117 | \$104 |
| Long-term debt | 471 | 458 |
| Total long-term liabilities | \$588 | \$562 |
| Stockholder's equity: |  |  |
| Preferred stock | \$39 | \$39 |
| Common stock (\$1 per value) | 55 | 32 |
| Capital surplus | 347 | 327 |
| Accumulated retained earnings | 390 | 347 |
| Less treasury stock | -26 | -20 |
| Total equity | \$805 | \$725 |
| Total liabilities and stockholder's equity | \$1,879 | \$1,742 |

## Notes:

1. Long-term debt rose by ( $\$ 471$ million- $\$ 458$ million) $\$ 13$ million. This is the difference between $\$ 86$ million new debt and $\$ 73$ million in retirement of old debt.
2. Treasury stock rose by $\$ 6$ million. This reflects the repurchase of $\$ 6$ million stock.
3. XYZ Corporation reports $\$ 43$ million in new equity. The company issued 23 million shares at a price of $\$ 1.87$. The par value of common stock is increased by $\$ 23$ million and capital surplus is increased by $\$ 20$ million.

## Income Statement of XYZ Corporation

| XYZ CORPORATION <br> Income Statement <br> 20X2 <br> (in \$ millions) |  |
| :--- | ---: |
|  |  |
| Total operating revenues | $\$ 2,262$ |
| Cost of goods sold |  |
| Selling, general, and administrative expenses | -1655 |
| Depreciation | -327 |
| Operating income <br> Other income | -90 |
| Earnings before interest and taxes | $\$ 190$ |
| Interest expense | 29 |
| Pretax income | $\$ 219$ |
| Taxes | -49 |
| Current: \$71 | $\$ 170$ |
| Deferred: \$13 | -84 |
| Net income |  |
| Retained earnings: | - |
| Dividends: |  |
|  |  |
|  |  |

## Notes:

1. There are 29 million shares outstanding. Earnings per share and dividends per share can be calculated as follows:

Earnings per share $=\frac{\text { Net Income }}{\text { Total share outstanding }}$
Earnings per share $=\frac{\$ 86}{\$ 29}$
Earnings per share $=\$ 2.97$ per share
Dividend per share $=\frac{\text { Dividend }}{\text { Total share outstanding }}$
Dividend per share $=\frac{\$ 43}{\$ 29}$
Dividend per share $=\$ 1.48$ per share

## Cash Flow from Operating Activities of XYZ Corporation

To calculate cash flow from operations, start with net income, add back noncash items like depreciation and adjust for changes in current assets and liabilities (other than cash).

| XYZ CORPORATION <br> Cash Flow from Operating Activities <br> 20X2 <br> (in \$ millions) |  |
| :--- | ---: |
| Operations Net Income | $\$ 86$ |
| Depreciation | 90 |
| Deferred Taxes | 13 |
| Changes in Assets and Liabilities | $(24)$ |
| Accounts Receivable | 11 |
| Inventories | 16 |
| Accounts Payable | 18 |
| Accrued Expenses | $(3)$ |
| Notes Payable | $(8)$ |
| Other | $\mathbf{\$ 1 9 9}$ |
| Total Cash Flow from Operations | $\boxed{ }$ |

## Cash Flow from Investing Activities of XYZ Corporation

\left.| XYZ CORPORATION |  |
| :--- | ---: |
| Cash Flow from Investing Activities |  |
| 20X2 |  |
| (in \$ millions) |  |$\right]$

## Cash Flow from Financing Activities of XYZ Corporation

\left.| XYZ CORPORATION |  |
| :--- | ---: |
| Cash Flow from Financing Activities |  |
| 20X2 |  |
| (in \$ millions) |  |$\right]$

## Cash Flow Statement of XYZ Corporation

The statement of cash flows is the addition of cash flows from operations, cash flows from investing activities, and cash flows from financing activities.

| XYZ CORPORATION Statement of Cash Flows 20X2 (in \$ millions) |  |
| :---: | :---: |
| Operations |  |
| Net Income | \$86 |
| Depreciation | 90 |
| Deferred Taxes | 13 |
| Changes in Assets and Liabilities |  |
| Accounts Receivable | (24) |
| Inventories | 11 |
| Accounts Payable | 16 |
| Accrued Expenses | 18 |
| Notes Payable | (3) |
| Other | (8) |
| Total Cash Flow from Operations | \$199 |
| Investing Activities |  |
| Acquisition of fixed assets | \$(198) |
| Sales of fixed assets | 25 |
| Total Cash Flow from Investing Activities | \$(173) |
| Financing Activities |  |
| Retirement of debt (includes notes) | \$(73) |
| Proceeds from long-term debt sales | 86 |
| Dividends | (43) |
| Repurchase of stock | (6) |
| Proceeds from new stock issue | 43 |
| Total Cash Flow from Financing | \$7 |
| Change in Cash (on the balance sheet) | \$33 |

## Significance of Financial Statements

- A good Working Knowledge of financial Statements is desirable simply because these statements are the primary means of communicating financial information both within and outside the firm


## External Uses of Statement Analysis

- Trade Creditors -- Focus on the liquidity of the firm.
- Bondholders -- Focus on the long-term cash flow of the firm.
- Shareholders -- Focus on the profitability and long-term health of the firm.


## Internal Uses of Statement Analysis

- Plan -- Focus on assessing the current financial position and evaluating potential firm opportunities.
- Control -- Focus on return on investment for various assets and asset efficiency.
- Understand -- Focus on understanding how suppliers of funds analyze the firm.


## Significance of Financial Statements

- The reason, we rely on accounting figures for much of our financial information is that we are almost always unable to obtain all of market information we want.
- The only meaningful yardstick for evaluation business decisions is whether or not they create economic value.
- Clearly, one important goal of the accountant is to report financial information to the user in a form useful for decision making.
- But the financial statements don't come with a user's guide.
- We will try to fill up this gap through learning a comprehensive analysis of financial statements.


## Standardized Financial Statements

- One obvious thing we want to do with a company's financial statements is to compare them to those of other.
- It is almost impossible to directly compare the financial statements for two companies because of differences in size. So we will try to standardize the financial statements.


## Common-Size Statements

- One very common and useful way of standardized comparison is to work with percentages instead of dollars.
- So, a standardized financial statement presenting all items in percentages is called a commonsize statement.
- Balance sheet items are shown as a percentage of total assets and income statement items as a percentage of sales.


## COMMON SIZE STATEMENTS

- One very common and useful way of standardized comparison is to work with percentages instead of dollars
- So, a standardized financial statement presenting all items in percentages is called a commonsize statement.
- Balance sheet items are shown as a percentage of total assets and income statement items as a percentage of sales.


## A2Z Inc. <br> Balance Sheet as of December 31 <br> (\$ in millions)

| Assets | 20X1 | 20X2 |
| :--- | :---: | :---: |
| $\underline{\text { Current Asset }}$ |  |  |
| Cash | $\$ 84$ | $\$ 98$ |
| Accounts Receivable | 165 | 188 |
| Inventory | $\underline{393}$ | $\underline{422}$ |
| Total Current Assets | 642 | 708 |

## Fixed Assets

| Net Plant and Equipment | $\underline{2,731}$ | $\underline{2,880}$ |
| :--- | ---: | ---: |
| Total Assets | $\underline{\underline{\$ 3,373}}$ | $\underline{\underline{\$ 3,588}}$ |
| Liabilities and Equity | $\mathbf{2 0 X 1}$ | $\mathbf{2 0 X} \mathbf{2 0}$ |

## Current Liabilities

Accounts Payable 312344
Notes Payable ..... $231 \quad 196$
Total Current Liabilities ..... 543 ..... 540
Long-term Debts ..... 531 ..... 457
Stockholders' Equity

| Common Stock and Paid in surplus | 500 | 550 |
| :--- | ---: | ---: |
| Retained Earning | 1,799 | 2,041 |
| Total Stockholders' Equity | $\underline{2,299}$ | $\underline{2,591}$ |
| Total Liabilities and Equity | $\underline{\$ 3,373}$ | $\underline{\$ 3,588}$ |

## A2Z Inc., <br> Common-Size Balance Sheet

| Assets | 20X1 | 20X2 |
| :---: | :---: | :---: |
| Current Asset |  |  |
| Cash | 2.5\% | 2.7\% |
| Accounts Receivable | 4.9\% | 5.2\% |
| Inventory | 11.7\% | 11.8\% |
| Total Current Assets | 19.1\% | 19.7\% |
| Fixed Assets |  |  |
| Net Plant and Equipment | 80.9\% | 80.3\% |
| Total Assets | 100.0\% | 100.0\% |
| Liabilities and Equity | 20X1 | 20X2 |
| Current Liabilities |  |  |
| Accounts Payable | 9.2\% | 9.6\% |
| Notes Payable | 6.8\% | 5.5\% |
| Total Current Liabilities | 16.0\% | 15.1\% |
| Long-term Debts | 15.7\% | 12.7\% |
| Stockholders' Equity |  |  |
| Common Stock and Paid in surplus | 14.8\% | 15.3\% |
| Retained Earning | 53.3\% | 56.9\% |
| Total Stockholders' Equity | 68.1\% | 72.2\% |
| Total Liabilities and Equity | $\underline{\underline{100.0 \%}}$ | $\underline{\underline{100.0 \%}}$ |

## A2Z Inc., Common-Size Balance Sheet

More on Standardized Statements

- Suppose we ask: "What happened to A2Z's net plant and equipment (NP\&E) over the period?"
- Based on the 20 X 1 and $20 \mathrm{X} 2 \mathrm{~B} / \mathrm{S}$, NP\&E rose from $\$ 2,731$ to $\$ 2,880$, so NP\&E rose by $\$ 149$.
- Did the firm's NP\&E go up or down? Obviously, it went up, but so did total assets. In fact, looking at the standardized statements, NP\&E went from $80.9 \%$ of total assets to $80.3 \%$ of total assets.


## A2Z Inc., Common-Size Balance Sheet

More on Standardized Statements

- If we standardized the 20X2 numbers by dividing each by the 20 X 1 number, we get a common base year statement. In this case, $\$ 2,880 / \$ 2,731=1.0545$, so NP\&E rose by $5.45 \%$ over this period.
- If we standardized the 20X2 common size numbers by dividing each by the 20 X 1 common size number, we get a combined common size, common base year statement. In this case, $80.3 \% /$
$80.9 \%=99.26 \%$, so NP\&E almost remained the same as a percentage of assets.
- In absolute terms, NP\&E is up by $\$ 149$ or $5.45 \%$, but relative to total assets, NP\&E fell by 0.6\%.


## A2Z Inc., Common-Size Balance Sheet

More on Standardized Statements

- Current assets rose from $19.1 \%$ in 20X1 to $19.7 \%$ in 20X2
- Current liabilities declined from $16.0 \%$ to $15.1 \%$ of total liabilities and equity over the same time.
- Total equity rose from $68.1 \%$ of total liabilities and equity to $72.2 \%$.
- Overall, A2Z's liquidity as measured by current assets compared to current liabilities, increased over the year. Also, A2Z's indebtness diminished as a percentage of total assets.
- So we may conclude that balance sheet as grown stronger.


## A2Z Inc., Income Statement <br> For the Year 20X2 <br> (\$ in millions)

| Net Sales |  | $\$ 2311$ |
| :--- | ---: | ---: |
| Cost of Goods Sold |  | 1344 |
| Depreciation | $\underline{276}$ |  |
| Earnings before interest and taxes | 691 |  |
| Interest | $\underline{141}$ |  |
| Taxable Income |  | 550 |
| Taxes |  | $\underline{187}$ |
| Net Income | $\underline{\$ 363}$ |  |
| Dividends | 121 |  |
| Retained Earnings | 242 |  |

## A2Z Inc., Common-Size Income Statement

| Net Sales |  | $100.0 \%$ |
| :--- | ---: | ---: |
| Cost of Goods Sold |  | $58.2 \%$ |
| Depreciation | $\underline{11.9 \%}$ |  |
| Earnings before interest and taxes | $29.9 \%$ |  |
| Interest | $\underline{6.1 \%}$ |  |
| Taxable Income |  | $23.8 \%$ |
| Taxes | $\underline{8.1 \%}$ |  |
| Net Income | $\underline{\underline{15.7 \%}}$ |  |
| Dividends | $5.2 \%$ |  |
| Retained Earnings | $10.5 \%$ |  |

## A2Z Inc., Common-Size Income Statement

- This statement tells us what happened to each dollar in sales.
- For A2Z interest expense eats up $6.1 \%$ of sales, while taxes take another $8.1 \%$ of sales figure.
- Following this, $15.7 \%$ of revenues from sales flow down to bottom as net income; one-third of which is paid in dividends and remainder two-thirds is taken as retained earnings for business.
- As far as cost is concerned, $58.2 \%$ of revenues are spent on the goods sold


## Standardized Financial Statements

- Although an organization's common-size statements provide a better analytical insight into its strength and standing, yet its performance and efficiency can be better judged by comparing these with those of the firm's competitors


## Ratio Analysis

- Another way of avoiding the problems involved in comparing companies of different sizes, is to calculate and compare financial ratios.
- One problem with ratios is that different people and different sources frequently don't compute them in exactly the same way.
- While using ratios as a tool for analysis, you should be careful to document how you calculate each one, and, if you are comparing your numbers to those of another source, be sure you know how their numbers are computed.


## Ratio Analysis

- For each of the ratios we discuss, several questions come to mind:
- How is it computed?
- What is it intended to measure, and why might we be interested?
- What is the unit of measurement?
- What might a high or low value be telling? How might such values be misleading?
- How could this measure be improved?


## Ratio Analysis

- Financial ratios are traditionally grouped into the following categories:
- Short-term solvency, or liquidity, ratios
- Ability to pay bills in the short-run
- Long-term solvency, or financial leverage, ratios
- Ability to meet long-term obligations
- Asset management, or turnover, ratios
- Intensity and efficiency of asset use
- Profitability ratios
- Ability to control expenses
- Market value ratios
- Going beyond financial statements


## Short-Term Solvency, or Liquidity, Measures

- The primary concern, to which these ratios relate, is the firm's ability to pay its bills over the short run without undue stress. So these ratios focus on current assets and current liabilities.
- Liquidity ratios are particularly interesting to short-term creditors. Since financial managers are constantly working with banks and other short-term lenders, an understanding of these ratios is essential
- Current assets and liabilities
- Their book values and market values are likely to be similar.
- They can and do change fairly rapidly, hence unpredictable


## Current Ratio

$$
\text { Current Ratio }=\frac{\text { Current Assets }}{\text { Current Liabilities }}
$$

- Because current assets and liabilities are converted into cash over the following 12 months, the current ratio is a measure of short run liquidity.
- The unit of measurement is either dollars or times.


## Current Ratio

- For A2Z Corporation, the 20X2 current ratio is:

$$
\text { Current Ratio }=\frac{\$ 708}{\$ 540}=1.31 \text { times }
$$

- We can say that
- A2Z has a $\$ 1.31$ in current assets for every $\$ 1$ in current liabilities OR
- A2Z has its current liabilities covered 1.31 times over.
- To a creditor (particularly a short-term creditor like supplier), the higher the current ratio, the better
- To firm, high current ratio indicates liquidity, but it may also indicate an inefficient use of cash and other short-term assets.
- We would expect to see a current ratio of at least 1 , because a current ratio of less than 1 would mean that net working capital is negative
- Like any other ratio, current ratio is affected by various transactions.
- If a firm borrows over long-term,
- The short run effect would be an increase in cash as well as in long term liabilities.
- Current liabilities would not be affected, so the current ratio would rise.
- An apparently low current ratio may not be a bad sign for a company with a large reserve of unlimited borrowing power.


## Current Events

- A firm wants to pay-off some of its suppliers and creditors. What would happen to current ratio?
- Current ratio moves away from 1. if it is greater than 1 it will get bigger. But if it is less than 1 , it will get smaller.
- Suppose a firm has $\$ 4$ in current assets and $\$ 2$ in current liabilities for a current ratio of 2 . and uses $\$ 1$ in cash to reduce current liabilities, then new current ratio is $(\$ 4-1) /(\$ 2-1)=3$
- Reversing the situation to $\$ 2$ in current assets and $\$ 4$ in current liabilities, the change will cause current ratio to fall to $1 / 3$ from $1 / 2$.
- Suppose a firm buys some inventory. What would happen in this case?
- Nothing happens to current ratio. Because in this scenario, one current asset (cash) goes down while another current asset (inventory) goes up. Total current assets are unaffected.
- What happens if a firm sells some merchandise?
- Current ratio would usually rise because inventory is shown at cost and sale would normally be at something greater than cost (difference is markup).
- So, the increase in either cash or receivables is greater than the decrease in inventory.
- This increases current assets and current ratio rises.


## Quick (or Acid-Test) Ratio

- Inventory is often the least liquid current asset. And its book values are least reliable as measures of market value since the quality of inventory isn't considered. Some of the inventory may turn out to be damaged, obsolete or lost.
- Relatively large inventories are often a sign of short-term trouble.
- The firm may have overestimated sales and overbought or overproduced as a result, hence tied up a substantial portion of its liquidity in slow moving inventory.

$$
\text { Quick Ratio }=\frac{\text { Current Assets - Inventory }}{\text { Current Liabilities }}
$$

- For A2Z, this ratio in 20X2 was:

$$
\text { Quick Ratio }=\frac{\$ 708-\$ 422}{\$ 540}=0.53 \text { times }
$$

The quick ratio here tells a somewhat different story than the current ratio, because inventory accounts for more than half of A2Z's current assets.

- If the same figure is for an aircraft manufacturing corporation, then this would certainly be cause for a BIG concern.


## Cash Ratio

- A very short-term creditor may be interested in the cash ratio.

$$
\text { Cash Ratio }=\frac{\text { Cash }}{\text { Current Liabilities }}
$$

- Current ratio for A2Z in 20X2 was 0.18


## RATIO ANALYSIS

## A2Z Inc., Balance Sheet <br> As of December 31 <br> (\$ in millions)

| Assets | 20X1 | 20X2 |
| :---: | :---: | :---: |
| Current Asset |  |  |
| Cash | \$84 | \$98 |
| Accounts Receivable | 165 | 188 |
| Inventory | 393 | 422 |
| Total Current Assets | \$642 | \$708 |
| Fixed Assets |  |  |
| Net Plant and Equipment | \$2,731 | \$2,880 |
| Total Assets | \$3,373 | \$3,588 |
| Liabilities and Equity | 20X1 | 20X2 |
| Current Liabilities |  |  |
| Accounts Payable | \$312 | \$344 |
| Notes Payable | $\underline{231}$ | 196 |
| Total Current Liabilities | \$543 | \$540 |
| Long-term Debts | 531 | 457 |
| Stockholders' Equity |  |  |
| Common Stock and Paid in surplus | \$500 | \$550 |
| Retained Earning | 1,799 | 2,041 |
| Total Stockholders' Equity | 2,299 | 2,591 |
| Total Liabilities and Equity | \$3,373 | \$3,588 |

## A2Z Inc., Income Statement For the Year 20X2 <br> (\$ in millions)

| Net Sales |  | $\$ 2,311$ |
| :--- | ---: | ---: |
| Cost of Goods Sold |  | 1,344 |
| Depreciation | $\underline{276}$ |  |
| Earnings before interest and taxes | $\$ 691$ |  |
| Interest | $\underline{141}$ |  |
| Taxable Income |  | 550 |
| Taxes | $\underline{187}$ |  |
| Net Income | $\underline{\$ 363}$ |  |
| Dividends | $\$ 121$ |  |
| Retained Earnings | $\$ 242$ |  |

## Current Ratio

## Current Events

- Suppose a firm buys some inventory. What would happen in this case?
- Nothing happens to current ratio. Because in this scenario, one current asset (cash) goes down while another current asset (inventory) goes up. Total current assets are unaffected.


## What happens if a firm sells some merchandise?

- Current ratio would usually rise because inventory is shown at cost and sale would normally be at something greater than cost (difference is markup).
- So, the increase in either cash or receivables is greater than the decrease in inventory.
- This increases current assets and current ratio rises.


## Current Ratio

- A firm wants to pay-off some of its suppliers and creditors. What would happen to current ratio?
- Current ratio moves away from 1. if it is greater than 1 it will get bigger. But if it is less than 1 , it will get smaller.
- Suppose a firm has $\$ 4$ in current assets and $\$ 2$ in current liabilities for a current ratio of 2 . and uses $\$ 1$ in cash to reduce current liabilities, then new current ratio is $(\$ 4-1) /(\$ 2-1)=3$
- Reversing the situation to $\$ 2$ in current assets and $\$ 4$ in current liabilities, the change will cause current ratio to fall to $1 / 3$ from $1 / 2$


## Long Term Solvency Measures

- These ratios are intended to address the firm's long-run ability to meet its obligations, or its financial leverage.


## Total Debt Ratio

This ratio takes into account all debts of all maturities to all creditors. It is computed as:

$$
\text { Total Debt Ratio }=\frac{\text { Total Assets -Total Equity }}{\text { Total Assets }}
$$

For A2Z Corporation,

$$
\text { Total Debt Ratio }=\frac{\$ 3,588-\$ 2,591}{\$ 3,588}=0.28 \text { times }
$$

- So, A2Z uses $28 \%$ debt. Whether this is high or low or whether it even makes any difference depends on whether or not capital structure matters.
- A2Z has $28 \%$ debt against total assets, thus there is $72 \%$ equity against total assets.
- Here we draw two variations out of total debt ratio
- Debt-equity ratio
- Equity multiplier
- Debt-Equity ratio = Total Debt / Total Equity
$=28 \% / 72 \%=0.39$ times
- Equity Multiplier $\quad=$ Total Assets / Total Equity
$=100 \% / 72 \%=1.39$ times
OR
$=1+$ Debt-Equity ratio $=1.39$ times


## Interest Coverage Ratio

- Also known as Times Interest Earned (TIE) ratio, refers to the ability of the firm to cover is interest obligations.

$$
\text { Interest Coverage Ratio }=\frac{\text { Earning Befor Interest and Taxes }}{\text { Interest }}
$$

For A2Z corporation,

$$
\text { Interest Coverage Ratio }=\frac{\$ 691}{\$ 141}=4.9 \text { times }
$$

## Cash Coverage Ratio

- A problem with Interest Coverage Ratio is that it is based on Earnings before Interest and Taxes (EBIT) which is not really a measure of cash available to pay interest.
- The reason is that depreciation, a non-cash expense has been deducted out. So we use:

$$
\text { Cash Coverage Ratio }=\frac{\text { EBIT }+ \text { Depreciation }}{\text { Interest }}
$$

For A2Z corporation,

$$
\begin{aligned}
& \text { Cash Coverage Ratio }=\frac{\$ 691+\$ 276}{\$ 141} \\
& \text { Cash Coverage Ratio }=\frac{\$ 967}{\$ 141}=6.9 \text { times }
\end{aligned}
$$

## Asset Management or Turnover Measures

The measures in this section are sometimes called Asset Utilization Ratios. These are intended to describe how efficiently or intensively a firm uses its assets to generate sales.

## Inventory Turnover Ratio

Inventory turnover can be calculated as:

$$
\text { Inventory turnover Ratio }=\frac{\text { Cost of Goods Sold }}{\text { Inventory }}
$$

For A2Z corporation,

$$
\text { Inventory turnover Ratio }=\frac{\$ 1,344}{\$ 422}=3.2 \text { times }
$$

So, A2Z sold off or turned over the entire inventory 3.2 times. As long as stock-out and foregoing sales situation doesn't arise, the higher this ratio is, the more efficiently inventory is being managed.

## Days' Sales in Inventory

If we know sales were turned over 3.2 times during the year, we can calculate easily how long it took to turnover on average.

$$
\text { Day's sales in Inventory }=\frac{365 \text { days }}{\text { Inventory Turnover }}
$$

For A2Z corporation,

$$
\text { Day's sales in Inventory }=\frac{365}{3.2}=114 \text { days }
$$

So, inventory stays for just less than 4 months before being sold or it would take 114 days to sell off current inventory.

## Receivables Turnover

Now we take a look on how fast we collect on the sales of inventory.

$$
\text { Receivables turnover }=\frac{\text { Sales }}{\text { Accounts Receivables }}
$$

For A2Z corporation,

$$
\text { Receivables turnover }=\frac{\$ 2,311}{\$ 188}=12.3 \text { times }
$$

So A2Z collected its outstanding credit accounts and re-loaned the money 12.3 times during the year. (Assuming all the sales are credit sales. If not, we use only credit sales for this ratio)

## Days' Sales in Receivables

$$
\text { Day's Sales in Receivables }=\frac{365 \text { days }}{\text { Receivable Turnover }}
$$

For A2Z corporation,

$$
\text { Day's Sales in Receivables }=\frac{365}{12.3}=30 \text { days }
$$

So A2Z collects on its credit sales in a month, or the firm has 30 days' worth of sales uncollected. This ratio is also called Average Collection Period.

## A Variation: Payables Turnover

It describes a how long does the firm take to pay its bills, and is computed as:

$$
\begin{aligned}
& \text { Payables Turnover }=\frac{\text { Cost of Goods Sold }}{\text { Accounts payables }} \\
& \text { Payables Turnover }=\frac{\$ 1,344}{\$ 344}=3.9 \text { times }
\end{aligned}
$$

So, days it took to turn-over the payables are:

$$
=\frac{365}{3.9}=94 \text { days }
$$

This figure is very significant to the current as well as potential creditors of A2Z.

## RATIO ANALYSIS (CONT...)

## A2Z Inc., Balance Sheet

As of December 31
(\$ in millions)

| Assets | 20X1 | 20X2 |
| :---: | :---: | :---: |
| Current Asset |  |  |
| Cash | \$84 | \$98 |
| Accounts Receivable | 165 | 188 |
| Inventory | 393 | 422 |
| Total Current Assets | \$642 | \$708 |
| Fixed Assets |  |  |
| Net Plant and Equipment | \$2,731 | \$2,880 |
| Total Assets | \$3,373 | \$3,588 |
| Liabilities and Equity | 20X1 | 20X2 |
| Current Liabilities |  |  |
| Accounts Payable | \$312 | \$344 |
| Notes Payable | $\underline{231}$ | 196 |
| Total Current Liabilities | \$543 | \$540 |
| Long-term Debts | 531 | 457 |
| Stockholders' Equity |  |  |
| Common Stock and Paid in surplus | \$500 | \$550 |
| Retained Earning | 1,799 | 2,041 |
| Total Stockholders' Equity | 2,299 | 2,591 |
| Total Liabilities and Equity | \$3,373 | \$3,588 |

## A2Z Inc., Income Statement For the Year 20X2 (\$ in millions)

Net Sales ..... \$2,311
Cost of Goods Sold ..... 1,344
Depreciation ..... $\underline{276}$
Earnings before interest and taxes ..... \$691
Interest ..... 141
Taxable Income ..... 550
Taxes ..... 187
Net Income ..... $\$ 363$
Dividends ..... \$121
Retained Earnings ..... \$242

## Total Asset turnover

$$
\text { Total Assets turnover }=\frac{\text { Sales }}{\text { Total Assets }}
$$

For A2Z corporation,

$$
\text { Total Assets turnover }=\frac{\$ 2,311}{\$ 3,558}=0.64 \text { times }
$$

In other words, for every dollar in assets, A 2 Z generated $\$ 0.64$ in sales.

## Capital Intensity Ratio

It is simply the reciprocal of total assets turnover, or:

$$
\text { Capital Intensity Ratio }=\frac{\text { Total Assets }}{\text { Sales }}
$$

- It is interpreted as the dollar investment in assets needed to generate $\$ 1$ in sales. Higher values represent capital intensive industries.
- For A2Z, this ratio is computed to be 1.56 , i.e. A2z has to invest $\$ 1.56$ in assets to get $\$ 1$ in sales.


## Profitability Measures

- In one form or the other, these ratios are intended to measure how efficiently the firm uses its assets and how efficiently the firm manages its operations.
- The focus in this group is on the bottom line - net income.


## Profit Margin

Every company, big or small, pays very close attention to their profit margin.

$$
\text { Profit Margin Ratio }=\frac{\text { Net Income }}{\text { Sales }}
$$

For A2Z Company,

$$
\text { Profit Margin Ratio }=\frac{\$ 363}{\$ 2,311}=15.7 \%
$$

- So in accounting sense, A2Z generates a little less than 16 cents in profit for every dollar in sales.
- Other Things being equal, a relatively high profit margin is obviously desirable, corresponding to low expenses vs. sales.
- But Other Things are not always equal!
- For example, lowering the sales price will usually increase unit sales but will normally cause profit margin to shrink. Total Profit of operating cash flow may go up or down.


## Return on Assets

Return on Assets (ROA) is a measure of profit per dollar of assets:

$$
\text { Return on Assets }=\frac{\text { Net Income }}{\text { Total Assets }}
$$

For A2Z,

$$
\text { Return on Assets }=\frac{\$ 363}{\$ 3,588}=10.12 \%
$$

## Return on Equity

- Return on equity (ROE) is a measure of how the stockholders fared during the year.
- Since benefiting the shareholders is the goal of corporation, ROE is a true bottom line measure of performance.

$$
\text { Return on Equity }=\frac{\text { Net Income }}{\text { Total Equity }}
$$

For A2Z,

$$
\text { Return on Equity }=\frac{\$ 363}{\$ 2,591}=14 \%
$$

Therefore, for every dollar in equity, A2Z generated 14 cents in profit. But this is correct in accounting terms only.

## ROA and ROE

- Because ROA and ROE are most widely used and commonly cited numbers, so it should be kept in mind that these are accounting rates of return.
- That is why these are called return on book assets and return on book equity
- ROE is sometimes called return on Net Worth


## Market Value Measures

- This group of measures is based, in part, on information not necessarily contained in financial statements, like market price per share.
- These measures can be calculated directly only for publicly traded companies.


## Earnings per Share

Assuming,

- A2Zhas 33 million shares outstanding and
- Stock sold for $\$ 88$ per share at the end of year.

So, the Earnings per Share (EPS) will be:

$$
\text { Earning Per Share }=\frac{\text { Net Income }}{\text { Shares Outstanding }}=\frac{\$ 363}{33}=\$ 11
$$

## Price-Earning Ratio

Price-eamings or PE ratio is defined as:

For A2Z,

$$
\text { Price-Earning Ratio }=\frac{\text { Price Per Share }}{\text { Earnings per share }}
$$

$$
\text { Price-Earning Ratio }=\frac{\$ 88}{\$ 11}=8 \text { times }
$$

- So A2Z shares sell for eight times earnings or it carries a PE multiple of 8 .
- Since PE ratio measures how much investors are willing to pay per dollar of current earnings, higher PEs are often taken to mean that the firm has significant prospects for future growth.
- If a firm had no or almost no earnings, its PE would probably be quite large; so careful interpretation is required.


## Book Value per share

Book Value is calculated as:

$$
\text { Book Value }=\frac{\text { Total equity }}{\text { No.of shares outstanding }}
$$

For A2Z,

$$
\text { Book Value }=\frac{\$ 2,591}{33}=\$ 78.5
$$

Since book value per share is an accounting number, it reflects historical costs.

## Market-to-Book ratio

It is defined as:

For A2Z,

$$
\text { Mark-to-Book Ratio }=\frac{\text { Market value per share }}{\text { Book value per share }}
$$

Mark-to-Book Ratio $=\frac{\$ 88}{\$ 78.5}=1.12$ times

- The market-to-Book ratio compares the market value of the firm's investment to their costs.
- A value less than 1 could mean that the firm has not been successful overall in creating value for its stockholders.


## The Du Pont Identity

- The difference between the two profitability measures, ROA and ROE, is the use of debt financing, or financial leverage.
- The relationship between these measures can be illustrated by decomposing ROE into its component parts.

Recall,

$$
\text { ROE }=\frac{\text { Net Income }}{\text { Total Equity }}
$$

Multiplying it by Assets / Assets (without changing anything)

$$
\begin{aligned}
& \text { ROE }=\frac{\text { Net Income }}{\text { Total Equity }}=\frac{\text { Net Income }}{\text { Total Equity }} \times \frac{\text { Assets }}{\text { Assets }} \\
& \text { ROE }=\frac{\text { Net Income }}{\text { Assets }} \times \frac{\text { Assets }}{\text { Total Equity }}
\end{aligned}
$$

So, we have expressed ROE as a product of two other ratios; ROA and the equity multiplier.

$$
\begin{aligned}
& \text { ROE }=\text { ROA } \times \text { Equity Multiplier } \\
& \text { ROE }=\text { ROA } \times(1+\text { Debt-Equity Ratio })
\end{aligned}
$$

## THE DU-PONT IDENTITY

- The difference between the two profitability measures, ROA and ROE, is the use of debt financing, or financial leverage.
- The relationship between these measures can be illustrated by decomposing ROE into its component parts.

Recall,

$$
\begin{aligned}
& \text { ROE }=\frac{\text { Net Income }}{\text { Total Equity }} \\
& \text { Multiplying it by Assets } / \text { Assets (without changing anything) } \\
& \text { ROE }=\frac{\text { Net Income }}{\text { Total Equity }}=\frac{\text { Net Income }}{\text { Total Equity }} \times \frac{\text { Assets }}{\text { Assets }} \\
& \text { ROE }=\frac{\text { Net Income }}{\text { Assets }} \times \frac{\text { Assets }}{\text { Total Equity }} \\
& \text { ROE }=\frac{\text { Net Income }}{\text { Assets }} \times \frac{\text { Assets }}{\text { Total Equity }}
\end{aligned}
$$

So, we have expressed ROE as a product of two other ratios; ROA and the equity multiplier

$$
\begin{aligned}
& \text { ROE }=\text { ROA } \times \text { Equity Multiplier } \\
& \text { ROE }=\text { ROA } \times(1+\text { Debt-Equity Ratio })
\end{aligned}
$$

- Looking back at A2Z:
- Debt-Equity Ratio $=0.39$
- ROA $=10.12 \%$
- while ROE calculated previously $=14 \%$
- Now, using the decomposition method:

$$
\text { ROE }=10.12 \% \times 1.39=14 \%
$$

We can further decompose ROE by multiplying the top and bottom by total sale:

$$
\text { ROE }=\frac{\text { Sales }}{\text { Sales }} \times \frac{\text { Net Income }}{\text { Assets }} \times \frac{\text { Assets }}{\text { Total Equity }}
$$

Rearranging a bit,


ROE $=$ Profit Margin $\times$ Total Assets Turnover $\times$ Equity Multiplier
This last Expression is called Du Pont Identity after the Du Pont Corporation, which popularized its use.

$$
\begin{aligned}
& \mathrm{ROE}=15.7 \% \times 0.64 \times 1.39 \\
& \mathrm{ROE}=14 \%
\end{aligned}
$$

- The Du Pont identity tells us that ROE is affected by three things:
- Operating efficiency (as measured by profit margin)
- Asset use efficiency (as measured by total assets turnover)
- Financial Leverage (as measured by equity multiplier)

Considering the Du Pont identity,

- It appears that a firm could leverage up its ROE by increasing its amount of debt
- This will only happen if ROA exceeds interest rate on debt.
- The decomposition of ROE is a convenient way of systematically approaching the financial statements analysis.
- If ROE is unsatisfactory by some measure, then Du Pont identity tells you where to start looking for the reasons.


## Dividend Payout

We have seen earlier that the net income is divided into two pieces.

- The first piece is cash dividend paid to stockholders
- Leftover amount is the addition to retained earnings
- A2Z's Net Income was $\$ 363$, of which $\$ 121$ was paid out in dividends. As a percentage:

$$
\begin{aligned}
& \text { Dividend Payout ratio }=\frac{\text { Cash Dividends }}{\text { Net Income }} \\
& \text { Dividend Payout ratio }=\frac{\$ 121}{\$ 363}=33 \frac{1}{3} \%
\end{aligned}
$$

## Retention Ratio

Anything A2Z does not pay out in form of dividends must be retained in the firm. So retention ratio is:

$$
\begin{aligned}
& \text { Retention ratio }=\frac{\text { Retained Earnings }}{\text { Net Income }} \\
& \text { Retention ratio }=\frac{\$ 242}{\$ 363}=66 \frac{2}{3} \%
\end{aligned}
$$

- So, A2Z retains two-thirds of its net income.
- The retention ratio is also known as the plowback ratio, as this is the amount which is plowed back into the business.


## Payout and Retention

Q: LMN Corporation pays out $40 \%$ of net income in form of dividends. What is its retention ratio?
A: If payout ratio is $40 \%$, retention ratio is: $1-40 \%=60 \%$
Q: If net income of LMN is $\$ 800$, how much did stockholders actually receive?
A: $\quad$ Dividends are $\$ 800 \times 40 \%=\$ 320$

## Internal and Sustainable Growth

- Firm's Return on Assets and Return on Equity are frequently used to calculate two additional numbers, both of which have to do with the firm's ability to grow.
- Investors and others are frequently interested in knowing how rapidly a firm's sales can grow.
- But the important thing to recognize is that if sales are to grow, assets have to grow as well, at least over the long run.
- Further if assets are to grow, then the firm must somehow obtain money to pay for the purchases.
- So, the growth has to be financed. And more so, a firm's ability to grow depends on its financing policies.
- A firm has two broad sources of financing:
- Internal financing simply refers to what the firm earns and subsequently plows back into the business.
- External financing refers to funds raised by either borrowing money or selling stock.


## Internal Growth Rate

A firm having a policy of internal financing, won't borrow funds and won't sell any new stock. Internal growth rate represents how rapidly the firm grows.

$$
\text { Internal Growth Rate }=\frac{R O A \times \mathrm{b}}{1-(R O A \times \mathrm{b})}
$$

Where ROA is return on assets and $b$ is the retention ratio

For A2Z this rate is:

$$
\begin{aligned}
& \text { Internal Growth Rate }=\frac{0.1012 \times \frac{2}{3}}{1-\left(0.1012 \times \frac{2}{3}\right)} \\
& \text { Internal Growth Rate }=7.23 \%
\end{aligned}
$$

## Sustainable Growth Rate

- If a firm only relies on the internal financing, then through time, its total debt ratio will decline, because assets will grow but total debt will remain the same (or even fall if some is paid off).
- Frequently, firms have a particular total debt ratio or equity multiplier that they view as optimal.
- With this in mind we now consider how rapidly a firm can grow if:
- it wishes to maintain a particular total debt ratio; and
- it is unwilling to sell new stock

Given these assumptions, the maximum growth rate that can be achieved is called the Sustainable Growth Rate:

$$
\begin{aligned}
& \text { Sustainable Growth Rate }=\frac{R O E \times \mathrm{b}}{1-(R O E \times \mathrm{b})} \\
& \text { Sustainable Growth Rate }=\frac{0.14 \times \frac{2}{3}}{1-\left(0.14 \times \frac{2}{3}\right)}
\end{aligned}
$$

Sustainable Growth Rate $=10.29 \%$
The reason for Sustainable Growth rate (10.29\%) being larger than internal growth rate $(7.23 \%)$ is that, as the firm grows, it will have to borrow additional funds if it has to maintain a constant debt ratio. This new borrowing is an extra source of financing in addition to internally generated funds, so A2Z can expand more.

## Determinants of Growth

- Using Du Pont identity, we saw that ROE can be decomposed into its various components
- Since ROE appears so prominently in determination of Sustainable growth rate, the factors determining ROE are also important determinants of growth.
- We know that:
- ROE = Profit Margin $\times$ Total Assets Turnover x Equity Multiplier
- Anything that increases ROE will increase the Sustainable growth rate. Increasing the retention ratio will have the same effect.
- So putting it all together, the firm's ability to sustain growth depends explicitly on the four factors:
- Profit Margin
- Total Assets Turnover
- Financial Policy
- Dividend Policy


## Profit margin

An increase in profit margin will increase the firm's ability to generate funds internally and thereby increase its sustainable growth.

## Total Assets Turnover

An increase in firm's total assets turnover increases the sales grow and thereby increase the sustainable growth rate. Increasing total assets turnover is the same thing as decreasing capital intensity.

## Financial Policy

An increase in the debt-equity ratio increases the firm's financial leverage. Since this makes additional debt financing available, it increases the sustainable growth rate.

## Dividend Policy

A decrease in the percentage of net income paid out as dividends will increase the retention ratio. This increase internally generated equity and thus increases internal and sustainable growth.

- The sustainable growth rate illustrates the explicit relationship between the firm's four major areas of concern:
- Operating efficiency (as measured by profit margin)
- Asset use efficiency (as measured by total assets turnover)
- Financial policy (as measured by the debt-equity ratio)
- Dividend policy (as measured by the retention ratio)

If sales are to grow at a rate higher than the sustainable growth rate, the firm must:

- increase profit margins,
- increase total assets turnover,
- increase financial leverage,
- increase earnings retention, or
- Sell new shares


## USING FINANCIAL STATEMENTS INFORMATION

- Now we take a look at some practical aspects of the financial statements analysis.
- Reasons for doing financial statements analysis
- Benchmarking the information
- Problems arising in the process


## Why Evaluate Financial Statements

- Primary reason for looking at the accounting information is that we don't have and can't expect to get market value information. But if we have such information, we will use it instead of accounting data.
- If there is a conflict between accounting and market data, market data would be preferred.


## Why Evaluate Financial Statements

- Financial statements analysis is an application of management by exception and boils down to comparing ratios for one business with some average or representative ratios.
- The ratios differing considerably from averages are studied further.


## Internal Uses

- Performance Evaluation
- Profit margin and return on equity
- Comparing the performance of different divisions
- Planning for the future
- Historical information used for generating projections
- Checking the realism of assumptions for the projections


## External Uses

- Customers:
- To evaluate the credit standing of a new customer
- Large customers would eye on the sustainability of the firm
- Suppliers:
- Evaluate the financial worth of the supplier
- Suppliers would be concerned about the creditworthiness of the firm


## External Uses

- Competitors
- Potential strength of the competitors in case of a new product launched by a firm
- Acquisition of new firms
- Identification of potential targets
- What to offer.


## Choosing a Benchmark

- Benchmarking is to establish a standard to follow for comparison.
- Some methods of benchmarking are:
- Time-Trend analysis
- Peer Group Analysis


## Time-Trend Analysis

- Based on the historical data of the firm
- If the current ratio of a firm is 2.4 for the recent financial statements, we may compare it with the current ratios for last 10 years.
- We may find that current ratio has declined over the years because of
- More efficient usage of current assets
- Change in the nature of business of the firm
- Change in business practices of the firm


## Peer Group Analysis

Identifying the firms

- competing in the same markets,
- Having similar assets,
- Operate in similar ways

Benchmarking:

- averages for this group of firms OR
- the top firms among the group


## Problems with Financial Statements Analysis

- No underlying theory to help identify the items or ratios to look at or to guide in establishing benchmark
- Very little help on value and risk
- Which ratios matter the most?
- What a high or low value might be?
- Firms with many diversified businesses
- Different accounting standards and procedures in different parts of the world


## Time Value of Money

- It refers to the fact that a dollar in hand today is worth more than a dollar promised at some time in future.
- The trade-off between money today and money later depends on, among other things, the rate one can earn by investing the money today for some interest income.


## Simple Interest vs. Compound Interest

Its most basic form, interest is calculated by multiplying principal (amount invested) by rate (\% of interest) multiplied by time (number of periods the interest is calculated). This is called simple interest.

$$
I=P \times r \times t
$$

Where
$\mathrm{P}=>$ principal amount
$\mathrm{r}=>$ interest rate
$\mathrm{t}=>$ time periods (years)
I $=>$ simple interest
However, if interest is left in the account to accumulate for a longer period (usually longer than one year), common practice requires that after interest is earned and credited for a given period, the new sum of principal + interest must now earn interest for the next period, etc. This is compound interest.

$$
\mathrm{I}=\mathrm{Px} \mathrm{r} \mathrm{t}^{\mathrm{t}}
$$

## Future Value

- It refers to the amount of money an investment will grow to over some period of time at some given interest rate.
- Alternatively, future vale is the cash value of an investment at some time in future.
- Simple Interest is calculated as:

$$
I=P \times r \times t
$$

A $\$ 1,000$ deposit at $8 \%$ per year for 3 years' simple interest:
$\mathrm{I}=1000 \times .08 \times 3=240$
A $\$ 1000$ deposit at $8 \%$ simple interest for three years earns $\$ 240$ interest.

- The future value (FV) of a simple interest calculation is derived by adding the original principal back to the interest earned.

$$
\$ 1,000+\$ 240=\$ 1,240
$$

Expressed as a formula:

$$
\begin{aligned}
& \mathrm{FV}=\mathrm{P}(1+\mathrm{rt}) \\
& \mathrm{FV}=1000+(1000 \times .08 \times 3)=1,240
\end{aligned}
$$

In the one-period case, the formula for FV can be written as:
$\mathrm{FV}=\mathrm{C}_{0} \times(1+\mathrm{r})$
Where $\mathrm{C}_{0}$ is cash flow today (time zero) and ' $\mathbf{r}$ ' is the appropriate interest rate.

## Future Value for a Lump Sum

If $\$ 100$ is invested at $10 \%$ interest rate, the future value of this $\$ 100$ in each proceeding year would be:

1. $\$ 110=\$ 100 \times(1+.10)$
2. $\$ 121=\$ 110 \times(1+.10)=\$ 100 \times 1.10 \times 1.10$

$$
=\$ 100 \mathrm{x}(1.10)^{2}
$$

$$
\text { 3. } \begin{aligned}
\$ 133.10 & =\$ 121 \times(1+.10)=\$ 100 \times 1.10 \times 1.10 \times . ~ \\
& =\$ 100 \times(1.10)^{3}
\end{aligned}
$$

## The Multi-period Case: Future Value

Generalizing the future value of an investment over many periods:
$\mathrm{FV}=\mathrm{C}_{0} \times(1+\mathrm{r}) \mathrm{t}$
Where, $C_{0}$ is cash flow at date 0 ,
$r$ is the appropriate interest rate, and
t is the number of periods over which the cash is invested. The expression $(1+\mathrm{r})^{\mathrm{t}}$ is the
future value interest factor.

## The Multi-period Case: Future Value

| Year | Beginning Amount | Interest Earned | Ending Amount |
| :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\$ 100.00$ | $\$ 10.00$ | $\$ 110.00$ |
| $\mathbf{2}$ | 110.00 | 11.00 | 121.00 |
| $\mathbf{3}$ | 121.00 | 12.10 | 133.10 |
| $\mathbf{4}$ | 133.10 | 13.31 | 146.41 |
| $\mathbf{5}$ | 146.41 | $\underline{14.64}$ | 161.05 |

Total Interest $\underline{\underline{61.05}}$

The Multi-period Case: Future Value
Future
Value(\$)


## Future Value Projections



Future Value and Compounding


Future Value Interest Factors (FVIF)

| Number of <br> periods | $\mathbf{5 \%}$ | $\mathbf{y y y y}$ | Interest Rates |  |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1 0} \boldsymbol{\%}$ | $\mathbf{1 5} \%$ | $\mathbf{2 0} \boldsymbol{\%}$ |  |  |
| 1 | 1.0500 | 1.1000 | 1.1500 | 1.2000 |
| 2 | 1.1025 | 1.2100 | 1.3225 | 1.4400 |
| 3 | 1.1576 | 1.3310 | 1.5209 | 1.7280 |
| 4 | 1.2155 | 1.4641 | 1.7490 | 2.0736 |
| 5 | 1.2763 | 1.6105 | 2.0114 | 2.4883 |

## FUTURE VALUE

- Usually simple interest is used in financial institutions for interest periods of less than one year.
- If the rate is expressed as an annual rate (normal practice), then the time period ( t ) must be a fraction of a year.
- By investing $\$ 10,000$ in an $8 \%, 90$-day certificate of deposit, total proceeds at the end of the CD period will be:
- $\quad \mathrm{FV}=(10,000)+(10,000 \mathrm{x} .08 \mathrm{x} 90 / 365)=\$ 10,197.26$

If you were to invest $\$ 10,000$ at 5-percent interest for one year, your investment would grow to \$10,500
$\$ 500$ would be interest $(\$ 10,000 \times .05)$
$\$ 10,000$ is the principal repayment $(\$ 10,000 \times 1)$
$\$ 10,500$ is the total due. It can be calculated as:
$\$ 10,500=\$ 10,000 \times(1.05)$
$\$ 10,000$ today is worth $\$ 10,500$ in one year, given that interest rate is $5 \%$

## Present Value

- It refers to the current value of the future cash flow discounted at the appropriate discount rate.
- In other words, the amount one would need to invest today at some pre-determined interest rate to get some desired amount in future is the present value of the desired money.
- Often, if a bank or other financial institution loans a sum for a short term, the lender will prefer to calculate the interest up front and loan out the discounted principal, or principal minus interest to be earned.
- The interest to be paid up front on a loan is called discount and the discounted principal, or the actual amount loaned is called the present value (PV)

$$
\mathrm{PV}=\frac{F V}{(1+r t)}
$$

- Repeating the discount basic formula (simple interest):

$$
\mathrm{PV}=\frac{F V}{(1+r t)}
$$

- Example: If the bank loans out $\$ 10,000$ for 90 days at $8 \%$ simple interest, the PV is:

$$
\begin{aligned}
\text { PV } & =10000 /[1+(.08)(90 / 365)] \\
& =10000 / 1.019726 \\
& =\$ 9,806.56
\end{aligned}
$$

## Present Value

Suppose you need $\$ 400$ to buy textbooks next year and you can earn $7 \%$ on your money. How much do you have to put up today? Now, ----------- Present value x $1.07=\$ 400$. Solving for present value:

$$
\text { Present Value }=\frac{\$ 400}{(1.07)}=\$ 373.83
$$

So investing $\$ 373.83$ (present value) at $7 \%$ will result in having $\$ 400$ (future value) in one year. If you were to be promised $\$ 10,000$ due in one year when interest rates are at 5 -percent, your investment be worth \$9,523.81 in today's dollars.

$$
\$ 9,523.81=\frac{\$ 10,000}{(1.05)}
$$

So, the amount that a borrower would need to set aside today to, be able to meet the promised payment of $\$ 10,000$ in one year is called the Present Value (PV) of $\$ 10,000$.Note that ------- $\$ 10,000=$ $\$ 9,523.81 \times(1.05)$

In the one-period case, the formula for PV can be written as:

- Where C1 is cash flow at date 1 and
- $r$ is the appropriate interest rate or discount rate.

$$
\mathrm{PV}=\frac{C_{1}}{1+r}
$$

## Present Value for Multiple Periods

- Calculating present value for multiple periods is quite similar in nature as was in case of future value.
- General formula for calculating present value of $C$ cash flow in $t$ periods time is:

$$
\mathrm{PV}=\mathrm{C} \times \frac{1}{(1+r)^{t}}
$$

- $1 /(1+\mathrm{r})^{\mathrm{t}}$ is used to discount a future cash flow, so it is called the discount factor Or present value interest factor (PVIF r,t)

Calculating the present value of a future cash flow to determine its worth today is commonly called discounted cash flow (DCF) valuation

| Present Value Interest Factors (PVIF) |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
| Years | $\mathbf{5} \%$ | Rate of Interest |  |  |
|  | $\mathbf{1 0 \%}$ | $\mathbf{1 5 \%}$ | $\mathbf{2 0 \%}$ |  |
| 1 | 0.9524 | 0.9091 | 0.8696 | 0.8333 |
| 2 | 0.9070 | 0.8264 | 0.7561 | 0.6944 |
| 3 | 0.8638 | 0.7513 | 0.6575 | 0.5787 |
| 4 | 0.8227 | 0.6830 | 0.5718 | 0.4823 |
| 5 | 0.7835 | 0.6209 | 0.4972 | 0.4019 |

## Present Value for Multiple Periods

- Do you want to be a millionaire? No problem!
- Suppose you are currently 21 years old, and can earn 10 percent on your money.
- How much you must invest today in order to accumulate $\$ 1$ million by the time you reach age 65 ?

First define the variables:

$$
\begin{array}{ll}
\mathrm{FV}=\$ 1 \text { million } & \mathrm{r}=10 \text { percent } \\
\mathrm{t}=65-21=44 \text { years } & \mathrm{PV}=?
\end{array}
$$

Set this up as a future value equation and solve for the present value:

$$
\begin{aligned}
& \$ 1 \text { million }=\mathrm{PV} \times(1.10) 44 \\
& \mathrm{PV}=\$ 1 \text { million } /(1.10) 44=\$ 15,091
\end{aligned}
$$

Of course, we've ignored taxes and other complications, but stay tuned - right now you need to figure out where to get $\$ 15,000$ !

## Present Value of \$1 for Different Periods and Rates

Present value of \$1 (\$)


## Present Value

How much would an investor have to set aside today in order to have $\$ 20,000$ five years from now if the current rate is $15 \%$ ?


## Present Value vs. Future Value

- What we called the present value factor is just the reciprocal of the future value factor.
- Future value factor $=(1+r)^{\mathrm{t}}$
- Present value factor $=1 /(1+r)^{t}$

If we let $\mathrm{FV}_{\mathrm{t}}$ stand for the future value after t periods, then the relationship between the future value and the present value is:

$$
\begin{aligned}
& \mathrm{PV} \times(1+\mathrm{r})^{\mathrm{t}}=\mathrm{FVt} \\
& \mathrm{PV}=\mathrm{FV}_{\mathrm{t}} /(1+\mathrm{r})^{\mathrm{t}}=\mathrm{FV}_{\mathrm{t}} \times\left[1 /(1+\mathrm{r})^{\mathrm{t}}\right]
\end{aligned}
$$

This is also known as basic present value equation.

## EVALUATING INVESTMENTS

- A company is considering the purchase of an asset for $\$ 335$, which may be sold for $\$ 400$ in three years. If the discounting factor is $10 \%$, is this a good investment?
- Since the company can invest $\$ 335$ elsewhere at $10 \%$, in three years, it would grow to:

$$
\begin{aligned}
\$ 335 \times(1+\mathrm{r})^{\mathrm{t}} & =\$ 335 \times 1.1^{3} \\
& =\$ 335 \times 1.331 \\
& =\$ 445.89
\end{aligned}
$$

- Since the proposed purchase of asset pays out only $\$ 400$, it is not as good as other investment alternatives
- Another way of looking at it is to calculate the present value of $\$ 400$ in three years at $10 \%$ :

$$
\begin{aligned}
& \$ 400 \times\left[1 /(1+\mathrm{r})^{t}\right] \\
& =\$ 400 / 1.1^{3} \\
& =\$ 400 / 1.331 \\
& =\$ 300.53
\end{aligned}
$$

This tells us that we only have to invest about $\$ 300$ to get $\$ 400$ in three years, not $\$ 335$

## What Rate Is Enough?

- Assume the total cost of a college education will be $\$ 50,000$ when a child enters college in 12 years.
The Parents have $\$ 5,000$ to invest today.
- What rate of interest must you earn on your investment to cover the cost of your child's education?

$$
\begin{aligned}
& \mathrm{FV} \quad=\mathrm{Cx}(1+\mathrm{r})^{\mathrm{t}} \\
& \$ 50,000=\$ 5,000 \mathrm{x}(1+\mathrm{r})^{12} \\
& (1+\mathrm{r})^{12}=\$ 50,000 / \$ 5,000 \\
& (1+\mathrm{r})^{12}=10 \\
& (1+\mathrm{r})^{1 / 12}=(10)^{1 / 12} \\
& 1+\mathrm{r}=1.2115 \\
& \mathrm{r}=1.2115-1 \\
& \mathrm{r}=.2115
\end{aligned}
$$

## Franklin: A Case Study

- Benjamin Franklin died on April 17, 1790. In his will, he gave 1,000 pounds sterling to Massachusetts and the city of Boston. He gave a like amount to Pennsylvania and the city of Philadelphia.
- The money was paid to Franklin when he held political office, but he believed that politicians should not be paid for their service.
- Franklin originally specified that the money should be paid out 100 years after his death and used to train young people.
- Later, however, after some legal wrangling, it was agreed that the money would be paid out 200
- years after Franklin's death in 1990.
- By that time, the Pennsylvania bequest had grown to about $\$ 2$ million; the Massachusetts bequest had grown to $\$ 4.5$ million. The money was used to fund the Franklin Institutes in Boston and Philadelphia.
- Assuming that 1,000 pounds sterling was equivalent to 1,000 dollars, what rate did the two states earn? (Note: the dollar didn't become the official U.S. currency until 1792.)
- For Pennsylvania, the future value is $\$ 2$ million and the present value is $\$ 1,000$. There are 200 years involved, so we need to solve for r in the following:

$$
\begin{aligned}
& \$ 1,000=\$ 2 \text { million } /(1+\mathrm{r})^{200} \\
& (1+r)^{200}=2,000.00
\end{aligned}
$$

Solving for $r$, the Pennsylvania money grew at about $3.87 \%$ per year. The Massachusetts money did better; check that the rate of return in this case was $4.3 \%$. Small differences can add up!

## Finding the Number of Periods

If we deposit $\$ 5,000$ today in an account paying $10 \%$, how long does it take to grow to $\$ 10,000$ ?

$$
\begin{aligned}
& \mathrm{FV}=\mathrm{C}_{0} \times(1+r)^{t} \\
& \$ 10,000=\$ 5,000 \times(1+0.10)^{t} \\
& \frac{\$ 10,000}{\$ 50,00}=(1+0.10)^{t} \\
& (1.10)^{t}=2 \\
& \ln (1.10)^{t}=\ln 2 \\
& t=\frac{\ln 2}{\ln (1.10)} \\
& t=\frac{0.6931}{0.0953} \\
& t=7.27 \mathrm{Years}
\end{aligned}
$$

## Finding the Number of Periods: Rule of 72

- For reasonable rates of return, the time it takes to double the money, is given approximately by

$$
\mathrm{t}=72 / \mathrm{r} \%
$$

- Continuing with the example, we have discount rate of $10 \%$, so:

$$
\mathrm{t}=72 / 10=7.2 \text { years }
$$

- This rule is fairly applicable to discount rates in $5 \%$ to $20 \%$ range.


## Finding the Number of Periods: An Example

- A company has been saving up to purchase an asset. Total cost will be $\$ 10$ million. The company has currently about $\$ 2.3$ million.
- How long it will have to wait,
- if it can earn $5 \%$ on the money?,
- if it can earn $16 \%$ on the money?
- At $5 \%$ the company will have to wait for a long time:

$$
\begin{aligned}
& \$ 2.3=\$ 10 / 1.05 t \\
& 1.05 t=4.35 \\
& t=30 \text { years }
\end{aligned}
$$

- At $16 \%$, it will make it in 10 years.


## Summarizing Time Value Calculations

## I. Symbols:

$$
\begin{array}{ll}
\text { PV } & \text { = Present value, what future cash flows are worth today } \\
\text { FVt } & \text { F Future value, what cash flows are worth in the future } \\
\mathrm{r} & \text { = Interest rate, rate of return, or discount rate per period } \\
\mathrm{t} & =\text { number of periods } \\
\mathrm{C} & =\text { cash amount }
\end{array}
$$

II. Future value of $C$ dollars invested at $r$ percent per period for $t$ periods:

$$
\begin{aligned}
& \mathrm{FV}_{\mathrm{t}}=\mathrm{Cx}(1+\mathrm{r})^{\mathrm{t}} \\
& \text { The term }(1+\mathrm{r})^{\mathrm{t}} \text { is called the future value factor. }
\end{aligned}
$$

## III. Present value of $C$ to be received in $t$ periods at $r$ percent per period:

$\mathrm{PV}=\mathrm{C} /(1+\mathrm{r})^{\mathrm{t}}$
The term $1 /(1+\mathrm{r})^{\mathrm{t}}$ is called the present value factor.
IV. The basic present value equation giving the relationship between present and future value is:

$$
\mathrm{PV}=\mathrm{FV}^{\mathrm{t}} /(1+\mathrm{r})^{\mathrm{t}}
$$

## Valuation of Multiple Cash Flows

- So far we have restricted our attention to either future value of a lump-sum present amount or the present value of some single future cash flows.
- Now onwards we will focus on valuation of multiple cash flows.


## Future Value with Multiple Cash Flows

- If you deposit $\$ 100$ today in an account at $8 \%$ and deposit another $\$ 100$ in one year, how much will you have in two years?
- At the end of first year, you will have $\$ 108$ plus the second $\$ 100$ you deposit, amounting to \$208.
By depositing this amount at $8 \%$, what you will get at the end of 2 nd year is:

$$
\$ 208 \times 1.08=\$ 224.64
$$

- This can be better explained by using a time line.
A. The time line

B. Calculating the future value


An alternate way to calculate the future value is to calculate the future value of each cash flow separately.

- The future value of first $\$ 100$ in 2 years at $8 \%$ is:

$$
\mathrm{FV}=\$ 100 \times 1.08^{2}=\$ 100 \times 1.1664=\$ 116.64
$$

- The 2 nd $\$ 100$ is deposited for one year, thus:

$$
\mathrm{FV}=\$ 100 \times 1.08=\$ 108
$$

- Adding up both the future values we get the total future value:

$$
\$ 116.64+\$ 108=\$ 224.64
$$

Future value calculated by compounding forward one period at a time


Future value calculated by compounding each cash flow separately


## Present Value with Multiple Cash Flows

- Suppose, we had an investment that was going to pay $\$ 1,000$ at the end of every year for next five years. The discount rate is assumed as $6 \%$.
- Present value of all these cash inflows can be calculated by:
- Discounting each cash flows separately OR
- Discounting back one period at a time.

Present value calculated by discounting each cash flow separately


Present value calculated by discounting back one period at a time


## How much is it worth?

- You are offered an investment that will pay you $\$ 200$ in one year, $\$ 400$ the next year, $\$ 600$ in 3 rd and $\$ 800$ at the end of 4th year.
- If you can earn $12 \%$ on very similar investments, what is the most you should pay for this one?
- Calculating the present value of each cash flow at $12 \%$ :

$$
\begin{aligned}
\$ 200 \times 1 / 1.12^{1} & =\$ 200 / 1.1200 \\
\$ 400 \times 1 / 1.12^{2}=\$ 400 / 1.2544 & =318.88 \\
\$ 600 \times 1 / 1.12^{3}=\$ 600 / 1.4049 & =427.07 \\
\$ 800 \times 1 / 1.12^{4} & =\$ 800 / 1.5735
\end{aligned}=508.41
$$

- If you can earn $12 \%$ on your money then you can duplicate this investment's cash flows for $\$ 1,432.93$. So, this is the most you should be willing to pay.


## Annuities and Perpetuities

- We will frequently encounter situations where we have multiple cash flows that are all the same amount.
- Series of equal installments for a loan-repayment
- A series of constant, or level, cash flows that occur at the end of each period for some fixed number of periods is called an ordinary Annuity.


## Present Value for Annuity cash flows

- For annuity calculation, we use a variation of present value equation.
- The present value of an annuity of $C$ dollars per period for $t$ periods when interest rate is $r$ is:

$$
\begin{aligned}
& \text { PV }=\mathrm{C} \times \frac{\{1-\text { Present value interest factor }\}}{r} \\
& \mathrm{PV}=\mathrm{C} \times \frac{\left\{1-\frac{1}{\left.(1+r)^{t}\right\}}\right.}{r}
\end{aligned}
$$

Where,
$\mathrm{C}=$ Periodic payment or annuity
$\mathrm{r}=$ rate of interest
$t=$ number of periods
The term in the parenthesis is called present value interest factor of an annuity ( $\mathrm{PVIFAr}, \mathrm{t}$ ).

## ANNUITIES

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$$

Where,
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$\mathrm{r}=$ rate of interest
$t=$ number of periods
The term in the parenthesis is called present value interest factor of an annuity.

## Present Value Interest Factor of an Annuity (PVIFA)

| Years | Rate of Interest |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{5 \%}$ | $\mathbf{1 0 \%}$ | $\mathbf{1 5 \%}$ | $\mathbf{2 0 \%}$ |
| $\mathbf{1}$ | 0.9524 | 0.9091 | 0.8696 | 0.8333 |
| $\mathbf{2}$ | 1.8594 | 1.7255 | 1.6257 | 1.5278 |
| $\mathbf{3}$ | 2.7232 | 2.4869 | 2.2832 | 2.1065 |
| $\mathbf{4}$ | 3.5460 | 3.1699 | 2.8550 | 2.5887 |
| $\mathbf{5}$ | 4.3259 | 3.7908 | 3.3522 | 2.9906 |

## How much can you afford?

- By looking at your budget you know you can pay $\$ 632$ for a new car, bank is offering you a loan for 48 months at $1 \%$ per month. How much should you borrow?

$$
\begin{aligned}
& \text { PVA }=C \times \frac{\{1-\text { Present value interest factor of Annuity }\}}{r} \\
& \text { PVA }=\$ 632 \times \frac{\{1-0.6203\}}{0.01} \\
& \text { PVA }=\$ 632 \times 37.9740 \\
& \text { PVA }=24,000
\end{aligned}
$$

- So, Present value $=\$ 632 \times 37.9740=\$ 24,000$
- Therefore, you can afford to borrow $\$ 24,000$


## Finding the Payment

- If you want to buy a new car costing $\$ 23,000$. With a $10 \%$ down payment, the bank will loan you the rest at $9 \%$ per year ( $.75 \%$ per month) for 60 months. How much will each monthly payment be?
- You will borrow $0.90 \times \$ 23,000=\$ 20,700$. This is the amount today, so it's the PV. The rate is $0.09 / 12=0.0075$, and there are 60 periods:

$$
\begin{aligned}
& \text { PVA }=\mathrm{C} \times \frac{\left\{1-\frac{1}{(1+r)^{t}}\right\}}{r} \\
& \$ 20,700=\mathrm{C} \times \frac{\left\{1-\frac{1}{(1+0.0075)^{60}}\right\}}{0.0075} \\
& \$ 20,700=\mathrm{C} \times 48.1737 \\
& C=\frac{\$ 20,700}{48.1737} \\
& C=\$ 429.70 \text { per month }
\end{aligned}
$$

## Finding the number of payments

- To repay a loan of $\$ 1,000$, Mr. X can only afford to pay $\$ 20$ per month. Interest rate is $1.5 \%$ per month.
- How long will it take to repay the loan?
- Here, $\mathrm{PV}=\$ 1,000, \mathrm{C}=\$ 20, \mathrm{r}=1.5 \%$ per month

$$
\begin{aligned}
& \$ 1000=\$ 20 \times(1-\mathrm{PVF}) / 0.015 \\
& (\$ 1,000 / 20) \times 0.015=1-\mathrm{PVF} \\
& \mathrm{PVF}=0.25=1 /(1+\mathrm{r}) \mathrm{t} \\
& 1.015 \mathrm{t}=1 / 0.25=4
\end{aligned}
$$

- So how long will it take to quadruple the money?
$1.015^{x}=4 \rightarrow 93$ month or 7.75 years
Calculation details
$1.015^{x}=4$
Taking $\log$ on both sides
$\log 1.015^{x}=\log 4$
$x \log 1.015=\log 4$
$x=\log 4 / \log 1.015$
$x=0.60206 / 0.00647$
$x=93$ months or 7.75 years

Finding the rate

- An insurance company offers to pay you $\$ 1,000$ per year if you pay $\$ 6,710$ up front. What rate is applicable in this 10-year annuity?
- Here,

$$
\begin{gathered}
C=\$ 1,000, \mathrm{PV}=\$ 6,710 \text { and } \mathrm{t}=10, \mathrm{r}=? \\
\$ 6,710=\$ 1,000 \times(1-\mathrm{PVF}) / \mathrm{r} \\
6.71=\{1-[1 /(1+\mathrm{r}) 10]\} / \mathrm{r}
\end{gathered}
$$

Looking at the PVIFA table for 10 periods, 6.7101 is the value for $8 \%$. So insurance company is offering $8 \%$

## Annuity Future Value

$$
\begin{aligned}
& \text { FVA }=\mathrm{C} \times \frac{\{\text { Future value interest factor of Annuity }-1\}}{r} \\
& \text { FVA }=C \times \frac{\left\{(1+r)^{t}-1\right\}}{r}
\end{aligned}
$$

## Future Value for Annuities

- Previously we determined that a 21 -year old could accumulate $\$ 1$ million by age 65 by investing $\$ 15,091$ today and letting it earn interest (at $10 \%$ compounded annually) for 44 years.
- Now, rather than plunking down $\$ 15,091$ in one chunk, suppose she would rather invest smaller amounts annually to accumulate the million.
- If the first deposit is made in one year, and deposits will continue through age 65, how large must they be?
- Set this up as a FV problem:

$$
\begin{aligned}
& \$ 1,000,000=C \times\left[(1.10)^{44}-1\right] / 0.10 \\
& C=\$ 1,000,000 / 652.6408=\$ 1,532.24 \\
& \text { Becoming a millionaire just got easier! }
\end{aligned}
$$

- Unfortunately, most people don't start saving for retirement that early in life. (Many don't start at all!)
- Suppose a 40 -year old person has decided it's time to get serious about saving. Assuming that he wishes to accumulate $\$ 1$ million by age 65 , he can earn $10 \%$ compounded annually, and will begin making equal annual deposits in one year, how much must each deposit be?
- Set this up as a FV problem:

$$
\begin{aligned}
& \mathrm{t}=65-40=25 \\
& \mathrm{FV}=\$ 1,000,000
\end{aligned}
$$

Then:

$$
\begin{aligned}
& \$ 1,000,000=C \times\left[(1.10)_{25}-1\right] / 0.10 \\
& C=\$ 1,000,000 / 98.3471=\$ 10,168.07
\end{aligned}
$$

- Moral of the story: Putting off saving for retirement makes it a lot more difficult!


## Annuities Due

- So far, we have discussed only ordinary annuities, where cash flows occur at the end of each period, e.g. loan repayments
- However, when you lease an asset, the first lease payment is usually due immediately, second at the beginning of second period and so on.
- An Annuity due is an annuity for which cash flows occur at the beginning of each period.
- The time line for Annuity due, having 5 payments of $\$ 400$ each, would be like:

- Present value of a four year $\$ 400$ ordinary annuity at $10 \%$ is $\$ 1,267.95$
- Adding on the extra $\$ 400$, we get $\$ 1,667.95$, the present value of this annuity due.
- The relationship between an annuity due and an ordinary annuity is just:

Annuity due value $=$ Ordinary annuity value $\mathrm{x}(1+\mathrm{r})$

## PREPETUITIES

- The present value of a perpetuity is
- Perpetuity PV $=\mathrm{C} / \mathrm{r}$
- Suppose we expect to receive $\$ 1000$ at the end of each of the next 5 years. Our opportunity rate is $6 \%$. What is the value today of this set of cash flows?

$$
\begin{aligned}
& \mathrm{PV}=\$ 1000 \times\left(1-\left[1 / 1.06^{5}\right]\right) / 0.06 \\
& \mathrm{PV}=\$ 1000 \times 4.212364 \\
& \mathrm{PV}=\$ 4212.364
\end{aligned}
$$

- Now suppose the cash flow will be $\$ 1000$ per year forever, making it perpetuity. In this case, the PV is easy to calculate:

$$
\mathrm{PV}=\mathrm{C} / \mathrm{r}=\$ 1000 / .06=\$ 16,666.67
$$

## Summary of Annuity and Perpetuity

## I. Symbols

PV = Present value, what future cash flows bring today
$\mathrm{FV}_{\mathrm{t}}=$ Future value, what cash flows are worth in the future
$r=$ Interest rate, rate of return, or discount rate per period
$t=$ Number of time periods
C = Cash amount
II. FV of $C$ per period for $t$ periods at $r$ percent per period:

$$
\mathrm{FVA}=\mathrm{C} \times \frac{\left\{(1+r)^{t}-1\right\}}{r}
$$

III. PV of $C$ per period for $t$ periods at $r$ percent per period:


## IV. $P V$ of perpetuity of $C$ per period:

$$
\mathrm{PV}=\mathrm{C} / \mathrm{r}
$$

## Effective Annual Rates

- If a rate is quoted as $10 \%$ compounded semiannually, then what this means is that the investment actually pays $5 \%$ every six months.
- Is $5 \%$ every six months the same thing as $10 \%$ per year?
- $\$ 1 \times 1.10=\$ 1.10$
- $\$ 1 \times 1.05^{2}=\$ 1.1025$
- $10 \%$ compounded semiannually is equivalent to $10.25 \%$ compounded annually.
- $10.25 \%$ is called effective annual rate (EAR)
- Suppose three banks offer you the following rates for a savings account:
- Bank A: $15 \%$, compounded daily
- Bank B: $15.5 \%$, compounded quarterly
- Bank C: $16 \%$, compounded annually
- Which one would you opt for:
- Bank C is offering $16 \%$ per year, since there is no compounding during the year, this is effective rate.
- Bank B actually paying $0.155 / 4=0.03875$ or $3.875 \%$ per quarter. An investment of $\$ 1$ at this rate for 1 year would grow to:

$$
\begin{aligned}
& \$ 1 \times 1.03875^{4}=\$ 1.1642 \\
& \text { So, EAR is } 16.42 \%
\end{aligned}
$$

- For saving, it is good. For borrowing it is worse
- Bank A is compounding daily, so the daily interest rate is actually $0.15 / 365=0.000411$ or $0.0411 \%$. An Investment of $\$ 1$ at this rate for 365 days would grow to:

$$
\begin{aligned}
& \$ 1 \times 1.000411365=\$ 1.1618 \\
& \text { So, EAR is } 16.18 \%
\end{aligned}
$$

- This is not as good as Bank B's $16.42 \%$ for a saver, and not as good as Bank C's $16 \%$ for a borrower
- Two Lessons:
- The highest quoted rate is not necessarily the best rate
- Compounding during the year can lead to a significant difference between the quoted rate and the effective rate
- EAR is computed in three steps
- Divide the quoted rate by the number of times the interest is compounded
- Add 1 and raise it to the power of number of times the interest is compounded.
- Subtract 1

So,
EAR $=(1+\text { Quoted rate } / m)^{m}-1$
Where $m$ is the number of times the interest is compounded
The EAR of $12 \%$ compounded monthly is:
EAR $=(1+\text { Quoted rate } / m)^{m}-1$
EAR $=(1+0.12 / 12)^{12}-1$
EAR $=(1.01)^{12}-1$
EAR $=1.12685-1$
$\mathrm{EAR}=12.6825 \%$

## Effective Annual Rate with changed number of compounding

| Compounding <br> period | Number of <br> times compounded | Effective <br> Annual Rate |
| :---: | :---: | :---: |
| Year | 1 | $10.000 \%$ |
| Quarter | 4 | 10.38129 |
| Month | 12 | 10.47131 |
| Week | 52 | 10.50648 |
| Daily | 365 | 10.51558 |
| Hour | 8,760 | 10.51703 |
| Minute | 525,600 | 10.51709 |

- A bank is offering $12 \%$ compounded quarterly. If you put $\$ 100$ in an account
- How much you will earn at the end of one year?
- What is the EAR?
- How much will you have at the end of two years?
- The bank is offering $12 \% / 4=3 \%$ every quarter, investing $\$ 100$ for 4 periods at $3 \%$ per period, the future value is:

$$
F V=\$ 100 \times 1.03^{4}=\$ 112.55
$$

The EAR is $12.55 \%$

- We can calculate the yield of two years in two ways
- Recognize two years as eight quarters:

$$
\$ 100 \times 1.03^{8}=\$ 126.68
$$

- Using EAR for two years:

$$
\$ 100 \times 1.1255^{2}=\$ 126.68
$$

- Anytime we do a present value or future value calculation, the rate we use must be an actual or effective rate.


## Annual Percentage Rates

- Annual percentage rate (APR) is the interest rate charged per period multiplied by the number of periods per year.
- Laws in some countries require that the lenders disclose an APR on virtually all the consumer loans in a prominent and un-ambiguous way.
- A typical credit card agreement quotes an interest tate of $18 \%$ APR. Monthly payments are required. What is the actual interest rate you pay on such a credit card ?
- APR of $18 \%$ with monthly payments is really $0.18 / 12=0.015$ or $1.5 \%$ per month.
- So,

$$
\begin{aligned}
& \mathrm{EAR}=(1+0.18 / 12)^{12}-1 \\
& \mathrm{EAR}=10.15^{12}-1=19.56 \%
\end{aligned}
$$

## Loans

- Whenever a lender extends a loan, some provision will be made for repayment of the principal amount.
- Here we discuss three forms of repayment of principle and interest patterns.
- Pure Discount Loans
- Interest-only Loans
- Amortized Loans


## Pure Discount Loans

- The borrower receives money today and repays a single lump sum at some time in the future.
- A borrower is able to repay $\$ 25,000$ in 5 years. Given a discount rate of $12 \%$, what amount of money the lender should lend?

$$
\begin{aligned}
& \text { Present value }=\$ 25,000 / 1.12^{5} \\
& \text { Present Value }=\$ 14,186
\end{aligned}
$$

## Interest-Only Loans

- Calls for the borrower to pay interest each period and to repay the entire principal at some time in the future.
- With a 3 -year, $10 \%$, interest-only loan of $\$ 1,000$, the borrower would pay $\$ 1,000 \times 0.10=\$ 100$ in interest at the end of first and second years,
- At the end of third year, he would return $\$ 1000$ plus the $\$ 100$ in interest for that year.


## Amortized Loans

- Amortized loan requires the borrower to repay parts of the loan amount over time.
- The process of paying off a loan by making regular principal deductions is called amortizing the loan. A loan can be amortized in two ways:
- Borrower pays the interest each period plus some fixed amount, e.g. medium-term business loans.
- Borrower makes a single fixed payment every period, car loans and mortgages.


## Amortization Schedule - Fixed Principal

A $\$ 5000$ loan at an interest rate of $9 \%$ for 5 years

| Year | Beginning <br> Balance | Total Payment | Interest Paid | Principal Paid | Ending <br> Balance |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\$ 5,000$ | $\$ 1,450$ | $\$ 450$ | $\$ 1,000$ | $\$ 4,000$ |
| $\mathbf{2}$ | 4,000 | 1,360 | 360 | 1,000 | 3,000 |
| $\mathbf{3}$ | 3,000 | 1,270 | 270 | 1,000 | 2,000 |
| $\mathbf{4}$ | 2,000 | 1,180 | 180 | 1,000 | 1,000 |
| $\mathbf{5}$ | 1,000 | $\underline{1,090}$ | $\underline{90}$ | $\underline{1,000}$ | 0 |
|  | Total |  |  |  | $\underline{\$ 6,350}$ |
| $\$ 1,350$ | $\underline{\$ 5,000}$ |  |  |  |  |

## Amortization Schedule - Fixed Payments

- Again, taking a $\$ 5,000$ loan at an interest rate of $9 \%$ for 5 years; we first need to determine the payment.
- Since this loan's cash flows are in the form of an ordinary annuity. So, solving for payments:

$$
\begin{aligned}
& \$ 5,000=\mathrm{C} x\left(1-1 / 0.09^{5}\right) / 0.09 \\
& \$ 5,000=\mathrm{Cx}(1-0.6499) / 0.09 \\
& \mathrm{C}=\$ 5,000 / 3.8897 \\
& \mathrm{C}=\$ 1,285.46
\end{aligned}
$$

## Amortization Schedule - Fixed Payments

| Beginning <br> Year | Total Balance | Total Payment | Interest <br> Paid | Principal Paid | Ending <br> Balance |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathbf{1}$ | $\$ 5,000$ | $\$ 1,285.46$ | $\$ 450$ | $\$ 835.46$ | $\$ 4,164.54$ |
| $\mathbf{2}$ | $4,164.54$ | $1,285.46$ | 374.81 | 910.65 | $3,253.89$ |
| $\mathbf{3}$ | $3,253.88$ | $1,285.46$ | 292.85 | 992.61 | $2,261.28$ |
| $\mathbf{4}$ | $2,261.27$ | $1,285.46$ | 203.51 | $1,081.95$ | $1,179.32$ |
| $\mathbf{5}$ | $1,179.32$ | $\underline{1,285.46}$ | $\underline{106.13}$ | $\underline{1,179.23}$ | 0 |
|  | Total | $\$ 6,427.30$ | $\$ 1,427.30$ | $\$ 5000$ |  |
|  |  |  |  |  |  |

## BONDS

- An evidence of debt issued by a corporation or a governmental body.
- When a corporation (or government) wishes to borrow from public on a long term basis, it does so by issuing or selling debt securities generally called bonds
- A bond represents a loan made by investors to the issuer. In return for his/her money, the investor receives a legal claim on future cash flows of the borrower.
- The issuer promises to:
- Make regular coupon payments every period until the bond matures, and
- Pay the face/par/maturity value of the bond when it matures.
- Default - since the abovementioned promises are contractual obligations, an issuer who fails to keep them is subject to legal action on behalf of the lenders (bondholders).
- B Corporation:
- Wants to borrow $\$ 1,000$ for 30 years at $12 \%$ interest rate
- Will pay $0.12 \times \$ 1,000=\$ 120$ in interest every year for 30 years.
- Will repay $\$ 1,000$ at the end of 30 years
- B Corporation
- $\$ 120$ regular interest payments are the bond's coupons
- $\$ 1,000$ is the par value or face value of the bond
- Annual coupon divided by the par value $(\$ 120 / \$ 1000=12 \%)$ is the coupon rate
- 30 years is the maturity time


## Bond Values and Yields

- The value of bonds may fluctuate as the interest rates change by time in the market place, though the cash flows from a bond remain the same.
- When interest rates rise, the present value of the bond's remaining cash flows decline and the bond is worth less
- When the interest rates fall, the bond is worth more.
- To determine the value of bond at a particular point in time, we need to know:
- No. of periods remaining till maturity,
- The face value,
- The coupon rate, and
- The market interest rate for similar bonds
- The interest rate required in the market on bonds is called the bond's Yield to Maturity
- The X Corporation issues a bond with 10 years to maturity having annual coupon of $\$ 80$. Similar bonds have a yield to maturity of $8 \%$.
- X bond's cash flows have two components:
$\bigcirc$ an annuity component (coupons) and
- a lump sum (face value paid at maturity)
- The X Corporation
- At the going interest rate of $8 \%$ the present value of $\$ 1,000$ paid in 10 years is:

$$
\mathrm{PV}=\$ 1,000 / 1.08^{10}=\$ 1,000 / 2.1589=\$ 463.19
$$

- Present value of the annuity of $80 \$$ per year for 10 years is:

$$
\begin{aligned}
& \mathrm{PV}=\$ 80 \times\left(1-1 / 1.08^{10}\right) / 0.08 \\
& \mathrm{PV}=\$ 80 \times 6.7101 \\
& \mathrm{PV}=\$ 536.81
\end{aligned}
$$

The X Corporation

- To get the bonds value we add up both parts

```
Total Bond Value \(=\$ 463.19+\$ 536.81\)
Total Bond Value \(=\$ 1,000\)
```

- This means that the bond sells for exactly its face value.

Alternatively,


- Interest rate change
- Interest rate risen to $10 \%$ after one years ( 9 years to maturity)
- Now the present value of $\$ 1,000$ paid in nine years at $10 \%$ is

$$
\$ 1,000 / 1.10^{9}=\$ 1,000 / 2.3579=\$ 424.10
$$

- And present value of $\$ 80$ annuity for 9 years at $10 \%$ is

$$
\$ 80 \times\left(1-1 / 1.10^{9}\right) / 0.10=\$ 80 \times 5.7590=\$ 460.72
$$

- Adding both parts:

$$
\text { Total bond value is } \$ 424.10+460.72=\$ 884.82
$$

- Therefore, the bond should sell for about \$885
- Because the bond sells for less than the going rate, investors are willing to lend something less than $\$ 1,000$.
- Because the bond sells for less than face value, it is said to be a discount bond.
- The investor who purchased and kept bond would get $\$ 80$ per year and would have a $\$ 115$ gain at maturity as well. This gain compensates the lender for below-market coupon rate.

Another way to see why bond is discounted by $\$ 115$ is to note that the $\$ 80$ coupon is $\$ 20$ below the coupon on a newly issued par value bond. So the investor who buys and keeps the bond gives up $\$ 20$ every year for 9 years. At $10 \%$ this annuity is worth:

$$
\$ 20 \times\left(1-1 / 1.10^{9}\right) / 0.10=\$ 20 \times 5.7590=\$ 115.18
$$

Just as rise of interest rates reflected a decline in the price of the bond, a drop of $2 \%$ in interest rates would result in the bond being sold for more than $\$ 1000$. Such a bond is said to sell at a premium or is called a premium bond.

## VALUING A BOND

- Assume you have the following information.
- BMN, Inc. bonds have a $\$ 1000$ face value
- The promised annual coupon is $\$ 100$
- The bonds mature in 20 years
- The market's required return on similar bonds is $10 \%$
- Present value of the face value

$$
\begin{aligned}
& =\$ 1000 \times\left[1 / 1.10^{20}\right] \\
& =\$ 1000 \times 0.14864 \\
& =\$ 148.64
\end{aligned}
$$

- Present value of the coupon payments

$$
\begin{aligned}
& =\$ 100 \times\left[1-\left(1 / 1.10^{20}\right)\right] / 0.10 \\
& =\$ 100 \times 8.5136 \\
& =\$ 851.36
\end{aligned}
$$

- The value of each bond $=\$ 148.64+851.36$

$$
=\$ 1000
$$

## Valuing a Bond: A Discount Bond

- Assume you have the following information.
- BMN, Inc. bonds have a $\$ 1000$ face value
- The promised annual coupon is $\$ 100$
- The bonds mature in 20 years
- The market's required return on similar bonds is $12 \%$
- Present value of the face value

$$
\begin{aligned}
& =\$ 1000 \times\left[1 / 1.12^{20}\right] \\
& =\$ 1000 \times 0.10366 \\
& =\$ 103.66
\end{aligned}
$$

- Present value of the coupon payments

$$
\begin{aligned}
& =\$ 100 \times\left[1-\left(1 / 1.12^{20}\right)\right] / 0.12 \\
& =\$ 100 \times 7.4694 \\
& =\$ 746.94
\end{aligned}
$$

- $\quad$ The value of each bond $=\$ 103.66+746.94$

$$
=\$ 850.60
$$

## Valuing a Bond: A Premium Bond

- Assume you have the following information.
- BMN, Inc. bonds have a $\$ 1000$ face value
- The promised annual coupon is $\$ 100$
- The bonds mature in 20 years
- The market's required return on similar bonds is $8 \%$
- Present value of the face value

$$
\begin{aligned}
& =\$ 1000 \times\left[1 / 1.08^{20}\right] \\
& =\$ 1000 \times 0.21455 \\
& =\$ 214.55
\end{aligned}
$$

- Present value of the coupon payments

$$
\begin{aligned}
& =\$ 100 \times\left[1-\left(1 / 1.08^{20}\right)\right] / 0.08 \\
& =\$ 100 \times 9.8181 \\
& =\$ 981.81
\end{aligned}
$$

- The value of each bond $=\$ 214.55+981.81$

$$
=\$ 1,196.36
$$

## Bond Price Sensitivity to YTM



## Valuing Bonds

Based on our examples we can write a general expression for the value of a bond. If a bond has

- a face value of $F$ paid at maturity
- a coupon of C paid per period
- $\quad \mathrm{t}$ periods to maturity
- a yield of r per period

Its value is:

$$
\text { Bond Value }=\mathrm{C} \times \frac{\left\{1-\frac{1}{(1+r)^{t}}\right\}}{r}+\frac{F}{(1+r)^{t}}
$$

Where,

- $\mathrm{C} \times \frac{\left\{1-\frac{1}{(1+\mathrm{r})^{\mathrm{t}}}\right\}}{\mathrm{r}}$ is the present value of coupons, and
- $\frac{\mathrm{F}}{(1+\mathrm{r})^{\mathrm{t}}}$ is the present value of face amount?


## Semiannual Coupons

- Bond yields are quoted like APRs, the quoted rate is equal to the actual rate per period multiplied by the number of periods in a year.
- When the payments of coupons are made on semiannually, each payment of half of the annual coupon, it is referred to as semiannual coupon bond
- If an ordinary bond has a coupon rate of $14 \%$ then the owner gets $\$ 140$ per year but in two installments each of $\$ 70$. in this case, the Yield to maturity is $16 \%$
- So with a $16 \%$ quoted yield, the true yield is $8 \%$ per period.
- Now if the bond has seven years to maturity
- What would be the bond's price?
- What is the effective annual yield on this bond?
- The bond should sell at a discount because it has a coupon rate of $7 \%$ per six months when the market requires $8 \%$ every six months.
- Present value of the face value

$$
\begin{aligned}
& =\$ 1000 / 1.08{ }^{14} \\
& =\$ 1000 / 2.9372 \\
& =\$ 340.46
\end{aligned}
$$

- Present value of 14 period annuity of $\$ 70$ at $8 \%$ discount rate is

$$
\begin{aligned}
& =\$ 70 \times\left[1-\left(1 / 1.08^{14}\right)\right] / 0.08 \\
& =\$ 70 \times 8.2442 \\
& =\$ 577.10
\end{aligned}
$$

- Total Present value tells that the bond should sell for:

$$
\begin{aligned}
& =\$ 340.46+577.10 \\
& =\$ 917.56
\end{aligned}
$$

- To calculate the effective annual yield on this bond, note that $8 \%$ every six months is equivalent to:

$$
\begin{aligned}
& \operatorname{EAR}=(1+0.08)^{2}-1 \\
& \operatorname{EAR}=16.64 \%
\end{aligned}
$$

## Interest Rate Risk

- The risk that arises for bond owners from fluctuating interest rates is called interest rate risk.
- The risk on a bond depends on how sensitive its price is to interest rate changes
- The sensitivity directly depends on
- Time to maturity
- Coupon rate
- While looking at a bond following should be kept in mind
- All other things being equal, the longer time to maturity, the grater the interest rate risk
- All other things being equal, the lower the coupon rate, the greater the interest rate risk
- We can illustrate these characteristics graphically


## Interest Rate Risk

| Interest Rate | Time to Maturity |  |
| :---: | :---: | :---: |
|  | 1 Year | $\mathbf{3 0}$ Years |
| $5 \%$ | $\$ 1047.62$ | $\$ 1768.62$ |
| 10 | 1000.00 | 1000.00 |
| 15 | 956.52 | 671.70 |
| 20 | 916.67 | 502.11 |

## Interest Rate Risk



- The slope of the line connecting the prices is much steeper for 30-year maturity than it is for 1 year maturity
- The steepness tells that a relatively small change in interest rate will lead to a substantial change in the bond's value.
- In comparison, one year bond is relatively less sensitive to interest rate changes
- The reason for longer term bonds having greater interest rate sensitivity is that a large proportion of the bond's value comes from the $\$ 1000$ face amount.
- The present value of this amount is not affected by small change in interest rate if amount is to be received in an year
- While, even a small change in interest rate when compounded for 30 years, will have a significant effect on the present value.
- Interest rate risk increases at a decreasing rate.
- If we compared a 10-year bond with a 1-year bond, obviously, 10 year bond has much greater risk
- But if we compare a 20 year bond to a 30 year bond, the 30 year bond will have a somewhat greater interest rate risk but the difference is fairly small.
- The bonds with lower coupons have greater interest rate risk
- If two bonds with different coupon rates have same maturity, the value of the one with the lower coupon is proportionately more dependent on the face amount to be received. So its value will fluctuate more as interest rate changes
- In other words, bond with higher coupon has a larger cash flow early in its life, so its value is less sensitive to changes in discount rate


## BOND PRICING THEOREMS

The following statements about bond pricing are always true:

- Bond prices and market interest rates move in opposite directions.
- When a bond's coupon rate is (greater than / equal to / less than) the market's required return, the bond's market value will be (greater than / equal to / less than) its par value.
- Given two bonds identical but for maturity, the price of the longer-term bond will change more than that of the shorter-term bond, for a given change in market interest rates.
- Given two bonds identical but for coupon, the price of the lower-coupon bond will change more than that of the higher-coupon bond, for a given change in market interest rates.


## Finding the Yield to Maturity

- We have seen that the price of a bond can be written as the sum of its annuity and lump sum components.
- Knowing that there is an $\$ 80$ coupon rate for 6 years and a $\$ 1000$ face value, the price of the bond is:

$$
\$ 955.14=\$ 80 \times\left[1-1 /(1+r)^{6}\right] / r+1000 /(1+r)^{6}
$$

- $r$ is the unknown discount rate or the yield to maturity
- To solve the above equation for r we will have to use trial and error method, as we cannot explicitly calculate $r$
- We can speed up the trial and error process by using our knowledge about prices and yields
- Since the bond is selling at a discount, we know that the yield is greater than $8 \%$.
- If we compute the price at $10 \%$, price is $\$ 912.89$ which is lower than the actual price, so $10 \%$ is too high. Rather it should be between $8 \%$ and $10 \%$
- Computing at $9 \%$ reveals that this is in fact the bond's yield to maturity.


## Summary of Bond Valuation

## I. Finding the value of a bond

$$
\begin{aligned}
& \text { Bond value }=\mathrm{C} \times[1-1 /(1+\mathrm{r}) \mathrm{t}] / \mathrm{r}+\mathrm{F} /(1+\mathrm{r}) \mathrm{t} \\
& \text { Where: } \mathrm{C}=\text { the promised coupon payment } \\
& \mathrm{F}=\text { the promised face value } \\
& \mathrm{t}=\text { number of periods until the bond matures } \\
& \mathrm{r}=\text { the market's required return, YTM }
\end{aligned}
$$

## II. Finding the yield on a bond

Given a bond value, coupon, time to maturity, and face value, it is possible to find the implicit discount rate, or yield to maturity, by trial and error only. To do this, try different discount rates until the calculated bond value equals the given bond value. Remember that increasing the rate decreases the bond value.

## Debt vs. Equity

- Securities issued by the corporations may be classified roughly a:
- Equity Securities
- Debt Securities
- When corporations borrow, they generally promise to
- Make regular scheduled interest payments, and
- Repay the original amount borrowed (principal)
- The main differences between debt and equity are the following:
- Debt is not an ownership interest in the firm. Creditors generally do not have voting power.
- Corporation's payment of interest on debt is considered as a cost of doing business and is fully tax deductible. While dividends paid to stockholders are not tax-deductible.
- Unpaid debt is a liability of the firm. If it is not paid, the creditors can legally claim the assets of the firm, resulting in bankruptcy or financial failure. This possibility does not arise when equity is issued.


## Long Term Debt

- Long term debt securities are promises made by the issuing firm to pay principal when due and to make timely interest payments on the unpaid balance.
- A number of features that distinguish the securities from one another
- Maturity is the length of time debt remains outstanding with some unpaid balance.
- Short-term debt (having maturity of one year or less) is sometimes referred to as unfunded debt
- Debt securities are typically called notes, debentures or bonds
- Strictly speaking a bond is a secured debt, but the word bond refers to all kinds of secure and unsecured debt.
- Public-issue bonds
- offered to general public
- Privately placed bonds
- placed with a private lender and not offered to general public


## Features of a Super Stores Bond

| Term Explanation |  |  |
| :---: | :---: | :---: |
| Amount of Issue | \$125 Million | The company will issue $\$ 125$ million worth of bonds. |
| Date of Issue | 28th Feb. 1986 | The bonds were sold on $28{ }^{\text {th }}$ Feb. 1986. |
| Maturity | $1^{\text {st }}$ April. 2016 | The principal will be paid in 30 years. |
| Face Value | \$1,000 | The denomination of the bonds is $\$ 1,000$. |
| Annual Coupon | 9.25\% | Each bondholder will receive $\$ 92.50$ per bond per year ( $9.25 \%$ of the face value). |
| Offer Price | \$1,000 | The offer price will be $100 \%$ of the $\$ 1,000$ face value per bond. |
| Coupon Payment | $1{ }^{\text {st }}$ Mar, $1^{\text {st }}$ Sep. | Coupons of $\$ 92.50 / 2=\$ 46.25$ will be dates paid on these dates. |
| Security | None | The bonds are debentures. |
| Sinking Fund | Annual | The firm will make annual payments toward the sinking fund. Beginning on 1st April, 1997 |
| Call Provision | Not Callable | The bonds have a deferred call feature before $28^{\text {th }}$ Feb. 1993 |
| Call Price | 106.48 Initially | After $28^{\text {th }}$ Feb. 1993, the company declining to 100 can buyback the bonds for $\$ 1,064.80$ per bond, declining to $\$ 1,000$ on $28^{\text {th }}$ Feb. 2005. |
| Rating | A2 | This is a higher rating. The bonds have a low probability of default. |

## THE BOND INDENTURE

- The bond indenture is a contract between the bond issuer and the bondholders. Usually, a trustee (perhaps a bank) is hired by the issuer to protect the bondholders' interests.
- The trust company must:
- Make sure the terms of indenture are obeyed
- Manage the sinking fund
- Represent the bondholders in default
- The indenture includes:
- The basic terms of the bond issue
- The total amount of bonds issued
- A description of the security
- The repayment arrangements
- The call provisions
- Details of the protective covenants


## Terms of a Bond

- Corporate bonds usually have a face value of $\$ 1000$, called principal value and it is stated on the bond certificate.
- The par value (initial accounting value) of a bond is almost always same as the face value, and terms are used interchangeably.
- The corporate bonds are usually in registered form.
- The company keeps a register recording the ownership of each bond and any changes thereof.
- The company will pay the interest and principal by cheque mailed directly to the address of the owner of record.
- The bond may be registered and have attached coupons. To obtain the interest payment, the owner must separate a coupon from the bond certificate and send it to company registrar
- Alternatively, the bond could be in bearer form, in which case, the certificate is the evidence of ownership and the company will pay the bearer. Ownership is not recorded, and the holder of the bond certificate detaches coupons and sends to company to receive payments.
- Bearer bonds are difficult to recover if lost or stolen.
- Company having no record of ownership cannot notify the bondholders of important events.


## Security

- Debt securities are classified as collateral and mortgages used to protect the bondholders
- Collateral means securities (bonds, stocks) or any asset pledged as security for payment of debt.
- Mortgage securities are secured by a mortgage on the real property of the borrower, involving usually real estate.
- Debenture is an unsecured bond for which no specific pledge of property is made
- The term note is used for such instruments if the maturity of the bonds is less than 10 years when issued


## Seniority

- Seniority indicates preference in position over lenders, and debts are sometimes labeled as senior or junior to indicate seniority.
- In the event of default, the holders of subordinated debt must give preference to other specified creditors. However, debt cannot be subordinated to equity


## Repayment

- Bonds can be repaid at maturity or they may be repaid in part or in entirety before maturity. Earlier repayment is handled through a sinking fund.
- A sinking fund is an account managed by the bond trustee for the purpose of repayment of bonds.
- The company makes annual payment to trustee who uses the funds to retire a portion of the debt, by either buying up some of the bonds in the market or, calling in a fraction of outstanding bonds
- Some types of sinking fund arrangements are:
- Some sinking funds start about 10 years after initial issuance
- Some sinking funds establish equal payments over the life of bond
- Some sinking funds are insufficient to redeem the entire issue of high-quality bonds, creating a possibility of a large "balloon payment" at maturity


## Call Provision

- A call provision allows the company to repurchase, or "call" part or all of the bond issue at stated prices over a specific period.
- Generally, the call price is above the bond's stated value (par value)
- The difference between the call price and the stated value is the call premium. The amount of premium, initially set equal to the annual coupon payment, becomes smaller over time and declines to zero as call date moves closer to time of maturity.
- Call provisions are not usually operative during the first part of a bond's life, making it less of a worry for bondholders.
- For example, a company may be prohibited from calling its bonds for the first 10 years, making the bond call protective. This is called a deferred call provisions.


## Protective Covenants

- A protective covenant is that part of the indenture or loan agreement that limits certain actions a company might wish to take during the term of the loan.
- These covenants can be classified into two types:
- Negative covenants
- Positive covenants
- A negative covenant limits or prohibits actions that company might take. For example:
- Limitation on the amount of dividend according to some formula
- Restrict pledging assets to other lenders
- Barring merger with another firm
- Restricting selling or leasing assets
- Barring issuance of additional long-term debt.
- A positive covenant specifies an action that the company agrees to take or a condition the company must abide by. For example:
- The firm must maintain its working capital at or above some specified minimum level
- The firm must periodically furnish audited financial statements to the lender
- The firm must maintain any collateral or security in good condition.


## Bond Ratings

- The bond ratings are an assessment of the creditworthiness of the corporate issuer.
- The definitions of creditworthiness used by the rating agencies are based on how likely the issuer firm is to default and the protection creditors have in the event of a default.
- These ratings are concerned only with the possibility of the default. Since they do not address the issue of interest rate risk, the price of a highly rated bond may be quite volatile.
- Long Term Ratings by PACRA
- AAA: Highest credit quality. 'AAA' ratings denote the lowest expectation of credit risk.
- AA: Very high credit quality. 'AA' ratings denote a very low expectation of credit risk.
- A: High credit quality. ' $A$ ' ratings denote a low expectation of credit risk.
- BBB: Good credit quality. ' BBB ' ratings indicate that there is currently a low expectation of credit risk.
- Long Term Ratings by PACRA

Speculative Grades:

- BB: Speculative. ' BB ' ratings indicate that there is a possibility of credit risk developing,
- B: Highly speculative. ' B ' ratings indicate that significant credit risk is present, but a limited margin of safety remains.
- CCC, CC, C: High default risk. Default is a real possibility.
- Short Term Ratings by PACRA
- A1+: highest capacity for timely repayment.
- A1: strong capacity for timely repayment.
- A2: satisfactory capacity for timely repayment may be susceptible to adverse economic conditions.
- A3: an adequate capacity for timely repayment. More susceptible to adverse economic conditions.
- Short Term Ratings by PACRA
- B: timely repayment is susceptible to adverse changes in business, economic, or financial conditions.
- C: an inadequate capacity to ensure timely repayment.
- D: high risk of default or which are currently in default.


## DIFFERENT TYPES OF BONDS

- Government Bonds
- Zero Coupon Bonds
- Floating-Rate Bonds
- Other Bonds


## Government Bonds

- When the government wishes to borrow money for more than one year, it sells what are known as treasury notes and bonds (mostly in the form of ordinary coupon bond) to the public.
- Govt. treasury issues have no default risk
- These issues are exempted from income taxes


## Zero Coupon Bonds

- A bond that pays no coupon at all and is offered at a price that is much lower than its stated value.
- Suppose N company issues a $\$ 1000$ face value, 5 year zero coupon bond.
- Initial price is set at $\$ 497$, yielding $15 \%$ to maturity
- Total interest paid over the life of the bond is $\$ 1000-497=\$ 503$
- For tax purposes, the issuer of a zero coupon bond deducts interest every year even though no interest is actually paid.
- Because of the tax break, the yields are lower than the yields on taxable bonds.


## Different Types of Bonds

Zero Coupon Bonds

| Year | Beginning Value | Ending Value | Implicit Interest <br> Expenses | Straight Line <br> Interest Expense |
| :---: | :---: | :---: | :---: | :---: |
| 1 | $\$ 497$ | $\$ 572$ | $\$ 75$ | $\$ 100.60$ |
| 2 | 572 | 658 | 86 | 100.60 |
| 3 | 658 | 756 | 98 | 100.60 |
| 4 | 756 | 870 | 114 | 100.60 |
| 5 | 870 | 1000 | $\underline{130}$ | $\underline{100.60}$ |
| Total |  |  | $\underline{\$ 503}$ |  |

## Floating Rate Bonds

- In case of floating rate bonds, the coupon payments are adjustable with respect to an interest rate index such as treasury bills interest rate.
- The value of a floating rate bond depends on this adjustment and its definition. In most cases, the coupon adjusts with a lag to some base rate.
- The majority of the floating rate bonds have the following features.
- Holder has the right to redeem the note at par on the coupon payment date after some specified period of time. This is called a put provision.
- Coupon rate has a floor and a ceiling i.e. coupon is subjected to a minimum and a maximum. Thus coupon rate is capped and upper and lower rates are the called the collar.
- An interesting type of floating rate bonds is an Inflation-linked bond.
- Such bonds have coupons that are adjusted according to the rate of inflation (the principal amount may be adjusted as well).


## Other types of Bonds

- Income bonds have coupon payments dependent on company income sufficient enough to support such payment
- Convertible bonds can be swapped for a fixed number of shares at any time before maturity at the holder's option.
- Put bonds allows the holder to force the issuer to buy the bond back at a stated price.


## Inflation and Interest Rates

- Real versus Nominal Rates
- Real rates are interest rates or rates of returns that have been adjusted for inflation
- Nominal rates are not adjusted for inflation
- Suppose rate of inflation is $5 \%$
- An investment costing $\$ 100$ today will be worth $\$ 115.50$ in one year, giving a rate of return of $15.5 \%$. But this $15.5 \%$ does not consider the effect of inflation, so this is nominal return
- Now suppose a pen costs $\$ 5$ a piece at the beginning of the year, giving 20 pens for $\$ 100$
- With inflation rate of $5 \%$, pens will cost $\$ 5.25$ at the end of the year.
- How many pens can be bought?
- What is rate of return?
- $\$ 115.50$ from the investment will buy us $\$ 115.50 / 5.25=22$ pens
- This is up from 20 pens, so pen rate of return is $10 \%$
- With nominal return of $15.5 \%$, our buying power goes up by only $10 \%$, because of inflation. So real rate of return is $10 \%$.
- Alternatively, with $5 \%$ inflation, each of the 115.50 nominal dollars we get is worth $5 \%$ less in real terms.
- Real value of our investment in an year is $\$ 115.50 / 1.05=\$ 110$
- giving a real rate of return of $10 \%$
- Real versus nominal returns:
- Your nominal return is the percentage change in the amount of money you have.
- Your real return is the percentage change in the amount of stuff you can actually buy.


## The Fisher Effect

The relationship between real and nominal returns is described by the Fisher Effect. Let:

$$
\begin{aligned}
& R=\text { the nominal return } \\
& r=\text { the real return } \\
& b=\text { the inflation rate }
\end{aligned}
$$

According to the Fisher Effect:

$$
1+\mathrm{R}=(1+\mathrm{r}) \mathrm{x}(1+\mathrm{h})
$$

From the example; the nominal rate is $15.5 \%$, and the inflation rate is $5 \%$ so real rate is

$$
\begin{aligned}
& 1+0.155=(1+r) x(1+0.05) \\
& (1+r)=1.155 / 1.05=1.10 \\
& r=10 \%
\end{aligned}
$$

Rearranging the fisher effect,

$$
\begin{aligned}
& 1+\mathrm{R}=(1+\mathrm{r}) \mathrm{x}(1+\mathrm{h}) \\
& \mathrm{R}=\mathrm{r}+\mathrm{h}+\mathrm{r} \times \mathrm{h}
\end{aligned}
$$

It tells that nominal rate has three components

- Real rate on investment r
- Compensation for the decrease in value of original investment because of inflation $h$
- Compensation for the decrease in value of income earned on investment due to inflation
- Dropping the third component (being very small), nominal rate gives us then approximately equal to:

$$
\mathrm{R} \approx \mathrm{r}+\mathrm{h}
$$

## TERM STRUCTURE OF INTEREST RATES

- The relationship between short- and long-term Interest rates
- Tells us what nominal interest rates are on default free, pure discount bonds of all maturities
- These are pure interest rates because they involve no risk of default and a single, lump-sum future payment
- When long term rates are higher than short term rates we say term structure is upward sloping and vice versa


## Determinants of Term Structure

- Real Rate of Interest
- Expected Inflation
- Interest Rate Risk


## Real Rate of Interest

- When real rate is high, all interest rates will tend to be higher and vice versa.
- Thus real rate does not really determine the shape of the term structure rather it influences the overall level of interest rates


## Prospect for Future inflation

- It very strongly influences the shape of the term structure.
- Value of dollar returns on investment for various periods of time may be eroded by future inflation.
- So investors demand a compensations for this loss in the form of Inflation premium (higher interest rates)
- Expectation of a higher inflation rate will push long term interest rates higher than short term rates reflected by an upward term structure.


## Interest Rate Risk

- Long term bonds have much greater risk of loss resulting from changes in interest rates than do short-term bonds.
- Investors recognize this risk and demand extra compensation in the form of higher rates for bearing it. This extra compensation is called the interest rate risk premium.
- The longer the term to maturity, the greater is the interest rate risk and the interest rate risk premium.
- Interest rate risk premium increases at a decreasing rate in line with the interest rate risk.


## The Term Structure of Interest Rate



## Bond Yields and the Yield Curve

We know that yields on treasury notes and bonds of different maturities are not the same. Plotting the treasury yields relative to maturity gives us a treasury yield curve (or just yield curve.)

## Bond Yields and the Yield Curve



- The shape of yield curve is a reflection of the term structure of the interest rates making these almost the same thing.
- The only difference is that the term structure is based on pure discount bonds whereas the yield curve is based on coupon bond yields.
- Treasury notes and bonds have three important features: they are:
- Default free
- Taxable
- Highly liquid
- Credit risk is the possibility of default.
- Investors demand a higher yield as compensation to the risk of possible default.
- This extra premium is called default risk premium
- We know that government bonds are free from most taxes, and have much lower yield than taxable bonds.
- Investors demand extra yield on a taxable bond as a compensation for the unfavorable tax treatment, known as taxability premium
- Bonds have varying degrees of liquidity.
- Due to a large number of bonds issued, you may not get as much good price if you want to sell quickly.
- Investor demand a liquidity premium for the compensation


## Common Stock Valuation

Valuation of a share of common stock is difficult due to:

- Not even promised cash flows are known in advanced
- Life of investment is forever, since common stock has no maturity
- Market rate of return is not easily observed


## Cash Flows

Suppose you buy a share of stock today, with a plan to sell it in a year, hoping that its worth will be $\$ 70$ at that time, along with a dividend payment of $\$ 10$ per share.

- If you require a $25 \%$ return on your investment, what is the most would you pay for the stock?
- Alternatively, what is the present value of the $\$ 10$ dividend along with the $\$ 70$ ending value at $25 \%$ ?
- The present value of the investment is
- Present value $=(\$ 10+\$ 70) / 1.25=\$ 64$
- So, $\$ 64$ is the value you would assign to the stock today.
- Generalizing this valuation, let
- $\mathrm{P}_{0}=>$ current price of stock
- $\mathrm{P}_{1}=>$ price in one period
- $\mathrm{D}_{1}=>$ Dividend paid at the end of the period
- So,

$$
P_{o}=\frac{\left(D_{1}+P_{1}\right)}{(1+R)}
$$

- Where R is the market rate of return
- But this possible only if we know P1, making the problem more complicated
- So, if we want to know the price in one year i.e. P1 and we somehow know the price in two years P 2 with D 2 dividend expected in two years, then

$$
\mathrm{P}_{1}=\frac{\left(D_{2}+P_{2}\right)}{(1+R)}
$$

- Now substituting this expression for P1 into our previous expression for P0, we would have the following equation:

$$
\begin{aligned}
& \mathrm{P}_{\mathrm{o}}=\frac{D_{1}+\frac{(D 2+P 2)}{(1+R)}}{(1+R)} \\
& \mathrm{P}_{\mathrm{o}}=\frac{D_{1}}{(1+R)^{1}}+\frac{D_{2}}{(1+R)^{2}}+\frac{P_{2}}{(1+R)^{2}}
\end{aligned}
$$

If we continue the substitution for 2 periods:

$$
\mathrm{P}_{\mathrm{o}}=\frac{D_{1}}{(1+R)^{1}}+\frac{D_{2}}{(1+R)^{2}}+\frac{D_{3}}{(1+R)^{3}}+\frac{P_{3}}{(1+R)^{3}}
$$

- Note that no matter what the stock price is, the present value is essentially zero if we push the sale of stock far enough away.
- So, the current price of stock can be written as the present value of the dividends beginning in one period and extending out forever
- Alternatively, we can say that the price of stock today is equal to the present value of all of future dividends.

$$
\mathrm{P}_{\mathrm{o}}=\frac{D_{1}}{(1+R)^{1}}+\frac{D_{2}}{(1+R)^{2}}+\frac{D_{3}}{(1+R)^{3}}+\frac{D_{4}}{(1+R)^{4}}
$$

## Zero Growth Stocks

- A share of common stock in a company with a constant dividend is termed as zero growth type of stocks, which implies:
$\mathrm{D} 1=\mathrm{D} 2=\mathrm{D} 3=\mathrm{D}=$ constant
- So the value of the stock is:

$$
\mathrm{P}_{\mathrm{o}}=\frac{D_{1}}{(1+R)^{1}}+\frac{D_{2}}{(1+R)^{2}}+\frac{D_{3}}{(1+R)^{3}}+\frac{D_{4}}{(1+R)^{4}}-\cdots
$$

- Since dividend is always the same, the stock can be viewed as an ordinary perpetuity with a cash flow equal to 'D' every period.
- So, the per share value is:

$$
\mathrm{P}_{\mathrm{o}}=\frac{D}{R}
$$

Where R is the required rate of return

- CVP Corporation has a policy of paying a $\$ 10$ per share dividend every year.
- If this policy is to continue indefinitely, what is the value of a share of stock if the required rate of return is $20 \%$ ?
- Since the stock amounts to be a perpetuity, value of the share is

$$
\$ 10 / 0.20=\$ 50 \text { per share }
$$

## ZERO GROWTH STOCKS

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$$
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$$

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$$
\mathrm{P}_{\mathrm{o}}=\frac{D_{1}}{(1+R)^{1}}+\frac{D_{2}}{(1+R)^{2}}+\frac{D_{3}}{(1+R)^{3}}+\frac{D_{4}}{(1+R)^{4}}-\cdots
$$

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$$
\$ 10 / 0.20=\$ 50 \text { per share }
$$

## Constant Growth Stocks

- Stocks, the dividend for which grows at a steady rate termed as growth rate g.
- If we let D 0 be the dividend just paid, then the next dividend, D 1 is

$$
\mathrm{D}_{1}=\mathrm{D}_{0} \mathrm{x}(1+\mathrm{g})
$$

- Dividend in two periods is :

$$
\begin{aligned}
& \mathrm{D}_{2}=\mathrm{D}_{1} \times(1+\mathrm{g}) \\
& \mathrm{D}_{2}=\left[\mathrm{D}_{0} \times(1+\mathrm{g})\right] \times(1+\mathrm{g}) \\
& \mathrm{D}_{2}=\mathrm{D}_{0} \times(1+\mathrm{g})^{2}
\end{aligned}
$$

- We can repeat this process to generalize it for any number of periods as

$$
D_{t}=D_{0} \times(1+g)^{t}
$$

- An asset with cash flows that grow at a constant rate forever is called a growing perpetuity
- The reason for the constant growth of dividend lies in the fact that the companies have this aspect as an explicit goal.
- The H Company has just paid a dividend of $\$ 3$ per share. The dividend of this company grows at a steady rate of $8 \%$ per year.
- Based on this information, what will be the dividend in 5 years?
- The future amount is

$$
\$ 3 \times 1.08^{5}=\$ 3 \times 1.4693=\$ 4.41
$$

- The dividend will therefore grow by $\$ 1.41$ over the coming years
- If dividend grow at a steady rate, we have replaced the problem of forecasting an infinite number of future dividends with the problem of coming up with a single growth rate.
- So, if we take $D_{0}$ to be the dividend just paid and $g$ to be the constant growth rate, the value of a share of stock can be written as:

$$
\begin{gathered}
\mathrm{P}_{\mathrm{o}}=\frac{D_{1}}{(1+R)^{1}}+\frac{D_{2}}{(1+R)^{2}}+\frac{D_{3}}{(1+R)^{3}}+\frac{D_{4}}{(1+R)^{4}}- \\
\mathrm{P}_{\mathrm{o}}=\frac{D_{o}(1+g)}{(1+R)^{1}}+\frac{D_{o}(1+g)^{2}}{(1+R)^{2}}+\frac{D_{o}(1+g)^{3}}{(1+R)^{3}}+\frac{D_{o}(1+g)^{4}}{(1+R)^{4}} \ldots
\end{gathered}
$$

As long as the growth rate, $g$, is less than the discount rate, $R$, the present value of cash flows can be written as:

$$
\mathrm{P}_{\mathrm{o}}=\frac{D_{o}(1+g)}{R-g}=\frac{D_{1}}{R-g}
$$

- This is commonly known as dividend growth model
- Suppose $\mathrm{D}_{0}$ is $\$ 2.30, \mathrm{R}$ is $13 \%$ and g is $5 \%$. The price per share in this case is:

$$
\begin{aligned}
& \mathrm{P}_{\mathrm{o}}=\frac{D_{O}(1+g)}{R-g} \\
& \mathrm{P}_{\mathrm{o}}=\frac{\$ 2.30(1.05)}{0.13-0.05} \\
& \mathrm{P}_{\mathrm{o}}=\frac{\$ 2.415}{0.08} \\
& \mathrm{P}_{\mathrm{o}}=\$ 30.19
\end{aligned}
$$

- We can use the dividend growth model to get the stock price at any point in time.
- The price of the stock as of time $t$ is:

$$
\mathrm{P}_{\mathrm{o}}=\frac{D_{t}(1+g)}{R-g}=\frac{D_{t+1}}{R-g}
$$

- Continuing with the previous example; D5 will be:

$$
\mathrm{D}_{5}=\$ 2.30 \times 1.05^{5}=2.30 \times 1.2763=\$ 2.935
$$

- From the dividend growth model, we get the price of the stock in 5 years

$$
\begin{aligned}
& \mathrm{P}_{5}=\frac{D_{5}(1+g)}{R-g} \\
& \mathrm{P}_{5}=\frac{\$ 2.935 \times 1.05}{0.13-0.05} \\
& \mathrm{P}_{5}=\frac{3.0822}{0.08} \\
& \mathrm{P}_{5}=\$ 38.5275
\end{aligned}
$$

- GG Company
- The next dividend of the company will be $\$ 4$ per share. Investors require a $16 \%$ return on the similar stocks. The company's dividend grows by $6 \%$ every year.
- Based on the dividend growth model
- What is the value of GG stock today?
- What is the value in four years?
- The only tricky thing is that the next dividend is $\$ 4$. so we won't multiply this by $(1+\mathrm{g})$. With this in mind the price per share is given by:

$$
\begin{aligned}
& \mathrm{P}_{\mathrm{o}}=\frac{D_{1}}{R-g} \\
& \mathrm{P}_{\mathrm{o}}=\frac{\$ 4}{0.16-0.06} \\
& \mathrm{P}_{\mathrm{o}}=\frac{\$ 4}{0.10} \\
& \mathrm{P}_{\mathrm{o}}=\$ 40
\end{aligned}
$$

- Since we already have dividend in one year, we know that the dividend in four years is equal to

$$
D_{1} \mathrm{x}(1+\mathrm{g})_{3}=\$ 4 \times 1.063=\$ 4.764
$$

- The price in four years is therefore:

$$
\begin{aligned}
\mathrm{P}_{4} & =\frac{D_{4}(1+g)}{R-g} \\
\mathrm{P}_{4} & =\frac{\$ 4.764 \times 1.06}{0.16-0.06} \\
\mathrm{P}_{4} & =\frac{\$ 5.05}{0.10} \\
\mathrm{P}_{4} & =\$ 50.50
\end{aligned}
$$

- Notice that

$$
\begin{aligned}
& \mathrm{P}_{4}=\mathrm{P}_{0 \times} \times(1+\mathrm{g})^{4} \\
& \mathrm{P}_{4}=\$ 40 \times 1.06^{4} \\
& \mathrm{P}_{4}=\$ 50.50
\end{aligned}
$$

- To see why this is so, notice that:

$$
\mathrm{P}_{4}=\frac{D_{5}}{R-g}
$$

- However, $\mathrm{D}_{5}$ is just equal to $\mathrm{D}_{1} \times(1+\mathrm{g})^{4}$, so we can write $\mathrm{P}_{4}$ as:

$$
\begin{aligned}
& \mathrm{P}_{4}=\frac{D_{1}(1+g)^{4}}{R-g} \\
& \mathrm{P}_{4}=\frac{D_{1}}{R-g} \times(1+g)^{4} \\
& \mathrm{P}_{4}=P_{0} \times(1+g)^{4}
\end{aligned}
$$

- This example illustrates that the dividend growth model makes the implicit assumption that the stock price will grow at the same constant rate as the dividend.
- What it tells us is that if the cash flows on an investment grow at a constant rate through time, so does the value of that investment.
- If the growth rate, $g$ is bigger than discount rate, $R$, then the present value of the dividends keeps on getting bigger and bigger. So the stock price is infinitely large.
- The same is true if growth rate and discount rate are equal.
- The expression we came up with for the constant growth case will work for any growing perpetuity, not just dividends on common stock.
- If C 1 is the next cash flow on a growing perpetuity, then the present value of the cash flows is given by:

$$
\mathrm{PV}=\frac{C_{1}}{R-g}=\frac{C_{0}(1+g)}{R-g}
$$

## Non-Constant Growth Stocks

Consider the case of a company that is currently not paying any dividends.

- You predict that in 5 years, the company will pay a dividend for the first time, say $\$ 0.50$.
- You expect that this dividend will grow at a rate of $10 \%$ per year indefinitely.
- The required rate of return on similar companies is $20 \%$.
- What is the price of the stock today?
- First we calculate what it will be worth once dividends are paid. We can then calculate the present value of that future price to get today's price.
- $1^{\text {st }}$ dividend will be paid in 5 years and it will grow steadily then onwards. So the price in 4 years will be:

$$
\begin{aligned}
& \mathrm{P}_{4}=\frac{D_{4}(1+g)}{R-g} \\
& \mathrm{P}_{4}=\frac{D_{5}}{R-g}=\frac{\$ 0.50}{0.20-0.10} \\
& \mathrm{P}_{4}=\$ 5
\end{aligned}
$$

- Now we can calculate the current value by discounting this price back four years at $20 \%$

$$
P_{0}=\$ 5 / 1.20_{4}=\$ 5 / 2.0736=\$ 2.41
$$

- The problem of non-constant growth is slightly complicated if the dividends are not zero for the first time.
- For example consider the following dividend forecasts for next three years.

| Year | Expected Dividend |
| :---: | :---: |
| 1 | $\$ 1.00$ |
| 2 | $\$ 2.00$ |
| 3 | $\$ 2.50$ |

- After 3rd year, dividend will grow at a constant rate of $5 \%$ per year
- The required return is $10 \%$. What is the value of the stock today?
- A time line can be quite helpful in dealing with such a problem of non-constant growth.

- First we compute the present value of the stock price three years down the road
- Then add in the present value of the dividends that will be paid between now and then.
- So the price in 3 years is:

$$
\begin{aligned}
& \mathrm{P}_{3}=\frac{D_{3}(1+g)}{R-g} \\
& \mathrm{P}_{3}=\frac{\$ 2.50 \times 1.05}{0.10-0.05} \\
& \mathrm{P}_{3}=\$ 52.50
\end{aligned}
$$

We can now calculate the total value of the stock as the present value of the first 3 dividends plus the present value of the price at time $3, \mathrm{P}_{3}$

$$
\begin{aligned}
& \mathrm{P}_{\mathrm{o}}=\frac{D_{1}}{(1+R)^{1}}+\frac{D_{2}}{(1+R)^{2}}+\frac{D_{3}}{(1+R)^{3}}+\frac{P_{3}}{(1+R)^{3}} \\
& \mathrm{P}_{\mathrm{o}}=\frac{\$ 1}{(1.10)^{1}}+\frac{\$ 2}{(1.10)^{2}}+\frac{\$ 2.50}{(1.10)^{3}}+\frac{\$ 52.52}{(1.10)^{3}} \\
& \mathrm{P}_{\mathrm{o}}=\$ 0.91+\$ 1.65+\$ 1.88+\$ 39.44 \\
& \mathrm{P}_{\mathrm{o}}=\$ 43.88
\end{aligned}
$$

## NON-CONSTANT GROWTH STOCKS

- CR Inc.: A Case of Supernormal Growth
- The company has been growing at a phenomenal rate of $30 \%$ per year. You expect that this growth rate to last for 3 more years and the rate will then drop to $10 \%$ per year.
- Total dividends just paid were $\$ 5$ million, and required return is $20 \%$
- If the growth rate then remains at $10 \%$ indefinitely, what is the total value of the stock?
- It is unlikely that a $30 \%$ supernormal growth rate can be sustained for any extended length of time.
- To value the equity in this company, we first need to calculate the total dividend over the supernormal growth period:

| Year | Total Dividend (in millions) |
| :---: | :--- |
| 1 | $\$ 5.00 \times 1.3=\$ 6.500$ |
| 2 | $\$ 6.50 \times 1.3=8.450$ |
| 3 | $\$ 8.45 \times 1.3=10.985$ |

The price at time 3 can be calculated as:

$$
\mathrm{P}_{3}=\mathrm{D}_{3} \times(1+\mathrm{g}) /(\mathrm{R}-\mathrm{g})
$$

Where ' $g$ ' is the long-run growth rate. So we have:

$$
P_{3}=\$ 10.985 \times 1.10 /(0.20-0.10)=\$ 120.835
$$

## Non-Constant Growth Stocks

To determine the value today, we need the present value of this amount plus the present value of the total dividends

$$
\begin{aligned}
& \mathrm{P}_{\mathrm{o}}=\frac{D_{1}}{(1+R)^{1}}+\frac{D_{2}}{(1+R)^{2}}+\frac{D_{3}}{(1+R)^{3}}+\frac{P_{3}}{(1+R)^{3}} \\
& \mathrm{P}_{\mathrm{o}}=\frac{\$ 6.50}{(1.20)^{1}}+\frac{\$ 8.45}{(1.20)^{2}}+\frac{\$ 10.985}{(1.20)^{3}}+\frac{\$ 120.835}{(1.20)^{3}} \\
& \mathrm{P}_{\mathrm{o}}=\$ 5.42+\$ 5.87+\$ 6.36+\$ 69.93 \\
& \mathrm{P}_{\mathrm{o}}=\$ 87.58 \text { (mill.) }
\end{aligned}
$$

## Components of Required Return

- Let's examine the implications of the dividend growth model for the required return of Discount rate R.
- We calculated $\mathrm{P}_{0}$ as

$$
\mathrm{P}_{0}=\mathrm{D}_{1} /(\mathrm{R}-\mathrm{g})
$$

- Rearranging it to solve for R , we get

$$
\begin{aligned}
& \mathrm{R}-\mathrm{g}=\mathrm{D} 1 / \mathrm{P} 0 \\
& \mathrm{R}=\mathrm{D} 1 / \mathrm{P} 0+\mathrm{g}
\end{aligned}
$$

- This tells us that the total return, R , has two components
- $\mathrm{D} 1 / \mathrm{P} 0$ is called the Dividend Yield. Because this is calculated as the expected cash dividend by the current price, it is conceptually similar to the current yield on a bond
- Growth rate, g , is also the rate at which the stock price grows. So it can be interpreted as capital gains yield
- Suppose a stock is selling for $\$ 20$ per share. The next dividend is $\$ 1$ per share and it is expected to grow by $10 \%$ more or less indefinitely. What return does this stock offer you if this is correct?
- The dividend growth model calculates the total return as:

$$
\begin{aligned}
& \mathrm{R}=\mathrm{D}_{1} / \mathrm{P}_{0}+\mathrm{g} \\
& \mathrm{R}=\$ 1 / 20+10 \%=5 \%+10 \%=15 \%
\end{aligned}
$$

- We can verify this answer by calculating the price in one year, P1, using $15 \%$ as the required return.
- Based on the dividend growth model, the price is:

$$
\begin{aligned}
& \mathrm{P}_{1}=\mathrm{D}_{1} \times(1+\mathrm{g}) /(\mathrm{R}-\mathrm{g}) \\
& \mathrm{P}_{1}=\$ 1 \times 1.10 /(0.15-0.10) \\
& \mathrm{P}_{1}=\$ 1.10 / 0.05 \\
& \mathrm{P}_{1}=\$ 22
\end{aligned}
$$

- This $\$ 22$ is equal to $\$ 20 \times 1.10$, so the stock price has grown by $10 \%$
- If you pay $\$ 20$ for the stock today, you will get a $\$ 1$ dividend at the end of the year, and you will have a $\$ 22-20=\$ 2$ gain
- Your dividend yield is thus $\$ 1 / 20=5 \%$, while your capital gains yield is $\$ 2 / 20=10 \%$
- So, total return would be $5 \%+10 \%=15 \%$


## Summary of Stock Valuation

The general Case

- The price today of a share of stock, P 0 , is the present value of all its future dividends, $\mathrm{D} 1, \mathrm{D} 2$, D3...

$$
\mathrm{P}_{\mathrm{o}}=\frac{D_{1}}{(1+R)^{1}}+\frac{D_{2}}{(1+R)^{2}}+\frac{D_{3}}{(1+R)^{3}}--
$$

- Where R is the required return

Constant Growth rate

- If the dividend is constant and equal to D , then the price is:

$$
\mathrm{P}_{0}=\mathrm{D} / \mathrm{R}
$$

- If the dividend grows at a steady rate, $g$, then the price will be:

$$
\mathrm{P}_{0}=\mathrm{D}_{1} /(\mathrm{R}-\mathrm{g})
$$

- This is called dividend growth model

Non-constant Growth rate

- The required return can be written as the sum of two things

$$
\mathrm{R}=\mathrm{D}_{1} / \mathrm{P}_{0}+\mathrm{g}
$$

- Where $\mathrm{D} 1 / \mathrm{P} 0$ is the dividend yield and g is the capital gains yield


## Common Stock Features

Stocks having no special preference either in paying dividend or in bankruptcy

- Shareholder Rights
- Proxy Voting
- Classes of Stock
- Other Rights
- Dividends


## Shareholder Rights

- Shareholders control the organization by electing the directors who, in turn, hire management to carry out their objectives.
- Directors are elected usually each year at an annual meeting through a golden rule of "one share - one vote."
- Either of two procedures is followed:
- Cumulative voting
- Straight voting


## Cumulative Voting

- Adopted to permit minority participation
- The total number of votes that each shareholder may cast is determined first by multiplying the number of shares with the number of directors to be elected.
- The top vote getters are all elected at once. Individual shareholders can distribute votes however they wish.
- For example, a corporation has two shareholders; Sami with 20 shares and Junaid with 80 shares.
- Both want to be a elected as one of the four director. But Junaid does not want Sami.
- Smith will cast $20 \times 4=80$ votes and Junaid will cast $80 \times 4=320$ votes.
- If Sami gives all his votes to himself, he is assured of a directorship. Because Junaid cannot divide 320 votes among four candidates in such a way as to give all of them more than 80 votes. So Sami will finish fourth at worst.
- Generally, $[1 /(\mathrm{N}+1) \%$ of stocks +1$]$ shares will guarantee you a seat.


## Straight Voting

- Directors are elected one at a time in this style.
- Applying to our example, Sami can cast 20 votes and Junaid can cast 80 . So Junaid will elect all the candidates.
- The only way to guarantee a seat is to own $50 \%$ plus one shares, also guaranteeing win over every seat, making it "all or nothing."


## Staggering

- Only a fraction of the directorship is up for election at a particular time.
- So if only two directors are up for election at any one time, it will take $1 /(2+1)=33.33 \%$ of the stocks plus one share to guarantee seat.
- It makes more difficult for a minority to elect a director when there is a cumulative voting because there are fewer directors to be elected at one time.
- It makes takeover attempts less likely to be successful since it makes it more difficult to vote in a majority of new directors.


## Proxy Voting

- A proxy is a grant of authority by a shareholder to someone else to vote the shareholder's shares.
- This is a routine practice in large corporations, since they have hundreds of thousands or even millions of shareholders.
- Obviously management would like to get as many proxies as possible transferred to it.
- However, an outside group may try to obtain votes via proxy if shareholders are dissatisfied with management.
- This may result in a proxy fight to retain or replace management by electing enough directors.


## Classes of Stocks

- Some firms have more than one class of common stocks with unequal voting rights
- In principle, stock exchange does not allow such classification.
- The primary reason for such stocks concerns with the control of the firm, as the firms can raise equity by issuing non-voting or limited-voting stocks while maintaining control.


## Other Rights

Some additional rights are:

- Right to share proportionately in dividends paid
- Right to share proportionately in assets remaining after liabilities have been paid in a liquidation
- Right to vote on matters of great importance, such as merger (usually done at annual general meeting or special meeting)
- Moreover, shareholders sometimes have the right to share proportionately in any new stock sold.
- This is called the preemptive right
- It means that a company that wishes to sell stock must first offer it to the existing shareholders before offering it to general public, so that they can protect their proportionate ownership in the firm.


## Dividends

- Dividends paid to the shareholders represent a return on the capital directly or indirectly contributed to the corporation by the shareholders.
- The payment of the dividend is at the discretion of the board of directors.
- Some important characteristics
- Unless a dividend is declared by the board of the directors of a corporation, it is not a liability of the corporation. The amount of the dividend and whether it is paid are decisions based on business judgment of the board of directors
- Payment of dividend is not a business expense and is not deductible for tax.
- Dividends received by individuals are considered ordinary income by tax authorities and are fully tax deductible. However, corporation owning stocks in other corporations are permitted to exclude certain percentage of dividend amount they receive and are taxed only on the remaining portion.


## Preferred Stock Features

- Stock with dividend priority over common stock, normally with a fixed dividend rate, sometimes without voting rights.
- Preference means only that the holders of the preferred shares must receive a dividend before holders if common shares are entitled to anything.


## Stated Value

- Preferred shares have stated liquidating value
- Cash dividend is described in terms of dollars per share.


## Accumulation of Dividend

- A preferred dividend is not like interest on a bond. The directors may decide not to pay the dividends on preferred stock irrespective of the net income of company.
- Dividends on preferred stock may be either cumulative or non-cumulative; most are cumulative
- If preferred stocks are cumulative and are not paid a particular year, they will be carried forward as an arrearage.
- Unpaid preferred dividends are not debts of the firm. Directors can defer the preferred dividend indefinitely. However, in this scenario, common stockholders must also forego dividends.
- Sometimes these delayed payments are compensated by voting rights.


## Is Preferred Stock Debt?

- Preferred shareholders are only entitled to receive a stated dividend and they are only entitled to the stated value of their shares
- Preferred stocks hold credit ratings much like bonds
- These are sometimes convertible into common stock and are often callable.


## The Stock Market

We know that the shares of stocks are bought and sold at stock exchanges.

- Stock market consists of:
- Primary market
- Secondary Market


## Primary Market

- The market in which new securities are originally sold to the investors.
- Companies sell securities to raise money.


## Secondary Market

- The market in which previously issued securities are traded among investors.


## Dealer

- An agent who buys and sells securities from a maintained inventory
- It stands ready to buy securities from investors wishing to sell them and sells securities to investors wishing to buy them
- The price that the dealer wishes to pay is the bid price and the price at which the dealer sells the securities is called the strike price.
- The difference between the bid and ask price is called the spread


## Broker

- An agent who arranges security transactions among investors, matching investors wishing to buy securities with investors wishing to sell securities.
- They do not buy or sell securities for their own accounts. Facilitating trades others is their business.


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## Capital Budgeting

What long-term investment should the firm take on?

## Net Present Value

- An investment is worth undertaking if it creates value for its owners characterized by worth in marketplace being more than what it costs to acquire.
- For example, you buy a run-down house for $\$ 25,000$ and spend another $\$ 25,000$ on painters, plumbers and so on to get it fixed up. So your total investment is $\$ 50,000$.
- When the work is completed, you place the house on the market and find that it is worth $\$ 60,000$.
- The market value $(\$ 60,000)$ exceeds the cost $(\$ 50,000)$ by $\$ 10,000$.
- If you consider yourself as manager and have brought together some fixed assets, labor and material etc., you have created $\$ 10,000$ in value.
- Now what you as a manager has to do is to identify the feasibility of investing $\$ 50,000$ ahead of time.
- In other words you are trying to determine whether a proposed investment or project will be worth more than it costs once it is in place, a topic we call Capital Budgeting
- The difference between an investment's market value and its cost is called the net present value of the investment (NPV).
- Alternatively, NPV is a measure of how much value is created or added today by undertaking an investment.
- Given our goal of creating value for our shareholders, the capital budgeting process can be viewed as a search for investment with positive net present values.
- Investment decisions are greatly simplified when there is a market for assets similar to the investment we are considering.
- Capital budgeting becomes more difficult when we can't determine the market price for comparable investment.
- The reason is that we are then faced with problem of estimating the value of an investment using only indirect market information.


## Estimating Net Present Value

## (\$000)



If we want to estimate the value of a new business, say fertilizer, we will

- Try to estimate the future cash flows we expect the new business to produce
- Apply discounted cash flow procedure to estimate the present value of the cash flows
- Estimate the difference between the present value of the future cash flows and the cost of investment
- This is known as discounted cash flow (DCF) valuation.
- Suppose the cash revenues from our fertilizer business will be $\$ 20,000$ per year.
- Cash costs (including taxes) will be $\$ 14,000$ per year
- The business will be wound up in 8 years
- The plant, property and equipment will worth $\$ 2,000$ as salvage value at that time.
- The project costs $\$ 30,000$ to launch
- Discount rate on similar projects is $15 \%$
- Is this a good investment?
- What will be the effect on price per share of 1,000 shares from taking the investment?
- Calculating the present value of the future cash flows:

$$
\begin{aligned}
& \mathrm{PV}=\$ 6,000 \times\left(1-1 / 1.15^{8}\right) / 0.15+2,000 / 1.15^{8} \\
& \mathrm{PV}=\$ 6,000 \times 4.4873+2,000 / 3.0590 \\
& \mathrm{PV}=\$ 26,924+654 \\
& \mathrm{PV}=\$ 27,578
\end{aligned}
$$

- Comparing this value with the $\$ 30,000$ estimated cost, the NPV is;

$$
\mathrm{NPV}=-\$ 30,000+27,578=-\$ 2,422
$$

- Therefore this is not a good investment, as it would decrease the total value of stock by $\$ 2,422$.
- With 1,000 shares outstanding, best estimate of impact of taking up this project is a loss of value of:

$$
\$ 2,422 / 1,000=\$ 2.422 \text { per share }
$$

- From this example we notice that if NPV is negative, the effect on share value will be unfavorable. If NPV were positive, the effect would be favorable.
- Given that the goal of financial management is to increase the share value, this discussion leads to the net present value rule


## "An investment should be accepted if the net present value is positive and rejected if it is negative."

- In the unlikely event that NPV is turned out to be zero, we would be indifferent between taking and not taking the investment
- Two comments
- The task of coming up with cash flows and the discount rate is much more important than the process of discounting itself.
- The process of discounting cash flows would only give us an estimated figure of NPV. The true NPV can be found by putting the investment for sale and see what we got for it.


## USING NPV

Given that the goal of financial management is to increase the share value, we have the net present value rule

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- The process of discounting cash flows would only give us an estimated figure of NPV. The true NPV can be found by putting the investment for sale and see what we got for it.
- Suppose we are asked to decide whether or not a new product be launched.
- Based on projected costs and sales, we expect that the cash flows over the 5 -year life of the project will be $\$ 2,000$ in first two years, $\$ 4,000$ in the next two and $\$ 5,000$ in the last year.
- It would cost about $\$ 10,000$ to begin production.
- Given a $10 \%$ discount rate, what should we do?
- The total value of the product by discounting its cash flow to present:

$$
\begin{aligned}
& \mathrm{PV}=\$ 2,000 / 1.1+2,000 / 1.1^{2}+4,000 / 1.1^{3}+4,000 / 1.1^{4}+5,000 / 1.1^{5} \\
& \mathrm{PV}=\$ 1,818+1,653+3,005+2,732+3,105 \\
& \mathrm{PV}=\$ 12,313
\end{aligned}
$$

- The present value of the expected cash flows is $\$ 12,313$, but the cost of getting those cash flows is $\$ 10,000$ so the NPV is $\$ 12,313-10,000=\$ 2,313$
- So, based on the NPV rule we should take on the project.


## The Payback Rule

The payback period is the amount of time required for an investment to generate cash flows sufficient to recover its initial cost.


- This investment pays for itself back in exactly two year OR payback period is two years.


## "An investment is acceptable if its calculated payback period is less than some specified number of years."

- Suppose the initial investment is $\$ 60,000$, and the cash flows are $\$ 20,000$ in the first year and $\$ 90,000$ in the second.
- The cash flows over the first two years are $\$ 110,000$, so the project pays back sometime in the second year.
- After the first year, the project has paid back $\$ 20,000$, leaving $\$ 40,000$ to be recovered
- Note that this $\$ 40,000$ is $\$ 40,000 / 90,000=4 / 9$ of the second year's cash flows.
- Spreading this ratio over 365 days:

$$
4 / 9 \times 365=162 \text { days (approx.) }
$$

This means, that the payback period is just over 1 year and 5 months.

- The Projected cash flows from a proposed investment are:

| Year | Cash Flow |
| :---: | :---: |
| 1 | $\$ 100$ |
| 2 | 200 |
| 3 | 500 |

- The project costs $\$ 500$. What is the payback period for this investment?
- After the first 2 years, the cash flows total $\$ 300$.
- After the 3 rd year, the total cash flow is $\$ 800$, so the project pays back sometimes between the end of year 2 and end of year 3 .
- Out of $\$ 500$ cash flows for 3rd year, we need to cover $\$ 200$, so we will have to wait $\$ 200 / 500$ $=0.40$ years to do this.
- The payback period is 2.4 years or about two years and five months.
- Based on this method of calculating payback, we can make decisions on investments.
- Decide on the cutoff time, say two years, for the investment
- Accept all the projects having payback of two years or less and reject all those having payback more than two years

| Year | A | B | C | D | E |
| :---: | ---: | ---: | ---: | ---: | ---: |
| 0 | $-\$ 100$ | $-\$ 200$ | $-\$ 200$ | $-\$ 200$ | $-\$ 50$ |
| 1 | 30 | 40 | 40 | 100 | 100 |
| 2 | 40 | 20 | 20 | 100 | $-50,000,000$ |
| 3 | 50 | 10 | 10 | -200 |  |
| 4 | 60 |  | 130 | 200 |  |

- Payback period of Project $A$ is 2.6 years.
- Project B never pays back.
- Project C has a payback period of 4 years.
- Project D has two payback periods; 2 and 4 years (both correct).
- Project E is unrealistic but it pays back in 6 months, thereby illustrating that a rapid payback does not guarantee a good investment.
- Compared to NPV, payback period rule has some shortcomings.
- Time value of money is ignored.
- It fails to consider risk differences.
- It doesn't provide an objective basis for a particular number for the cutoff period.
- Primary shortcomings of payback period rule:
- By ignoring time value, we may be led to take investments that are worth less than they cost.
- By ignoring cash flows beyond cutoff, we may be led to reject profitable long term investments.
- Generally, it tends to bias us towards shorter term investments
- Qualities of payback period rule:
- Simple and easy to calculate, useful for large number of small investment decisions by corporation
- Its biasness towards short term projects, emphasizes on liquidity, i.e. quickly freeing up cash for other uses
- It adjusts for more uncertain cash flows expected in later part of a project's life (though by ignoring them altogether)
- Consider the following investments:

| Year | Long | Short |
| :---: | :---: | :---: |
| 0 | $-\$ 250$ | $-\$ 250$ |
| 1 | 100 | 100 |
| 2 | 100 | 200 |
| 3 | 100 | 0 |
| 4 | 100 | 0 |

- The payback on long is

$$
2+\$ 50 / 100=2.5 \text { years }
$$

- The payback on short is

$$
1+\$ 150 / 200=1.75 \text { years }
$$

- With a cutoff of two years, short is accepted and long is not
- Is payback period rule giving us the right decisions?
- Suppose, we require a $15 \%$ return on this kind of investment; NPV for the two investments is:

NPV (Short) $=-\$ 250+100 / 1.15+200 / 1.15^{2}=-\$ 11.81$
NPV (Long) $=-\$ 250+100 \times\left(1-1 / 1.15^{4}\right) / .15=\$ 35.5$

- We can see that the NPV of shorter term investment is negative, which would diminish the value of shareholders' equity.
- Longer term investment increases share value.


## Summary

- A break-even measure; in accounting sense but not in economic sense.
- Does not focus on right issue of impact of investment on value of stock.
- Simplicity helpful for decisions on minor investments.


## Advantages

- Easy to understand
- Adjusts for uncertainty of later cash flows
- Biased towards liquidity


## Disadvantages

- Ignores the time value of money
- Requires an arbitrary cutoff point
- Ignores cash flows beyond the cutoff date
- Biased against long term projects, such as research and development and new projects


## AVERAGE ACCOUNTING RETURN

It is defined as:

$$
\mathrm{AAR}=\frac{\text { Some Measure of Average Accounting Profit }}{\text { Some Measures of Avereage Accounting Value }}
$$

Specifically:

$$
\mathrm{AAR}=\frac{\text { Average Net Income }}{\text { Average Book Value }}
$$

Suppose we are deciding whether or not to open a store in a new shopping center.

- The required investment in improvements is $\$ 500,000$
- The store has a 5-years life, as everything reverts to the owners of the shopping center after that time.
- The required investment would be $100 \%$ depreciated over 5 years, i.e.
- $\$ 500,000 / 5=\$ 100,000$ per year
- Tax rate is $25 \%$


## Average Accounting Return

| Particulars | Year 1 | Year 2 | Year 3 | Year 4 | Year 5 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Revenues | $\$ 433,333$ | $\$ 450,000$ | $\$ 266,667$ | $\$ 200,000$ | $\$ 133,333$ |
| Expenses | 200,000 | 150,000 | 100,000 | 100,000 | 100,000 |
| E.B.D | 233,333 | 300,000 | 166,667 | 100,000 | 33,333 |
| Depreciation | 100,000 | 100,000 | 100,000 | 100,000 | 100,000 |
| E.B.T | 133,333 | 200,000 | 66,667 | 0 | $-66,667$ |
| Taxes 25\% | 33,333 | 50,000 | 16,667 | 0 | $-16,667$ |
| Net Income | $\mathbf{1 0 0 , 0 0 0}$ | $\mathbf{1 5 0 , 0 0 0}$ | $\mathbf{5 0 , 0 0 0}$ | $\mathbf{0}$ | $\mathbf{- 5 0 , 0 0 0}$ |

- Average net income $=[\$ 100,000+150,000+50,000+0+(-50,000)] / 5$

$$
=\$ 50,000
$$

- Average book value $=(\$ 500,000+0) / 2$

$$
=\$ 250,000
$$

- The average accounting return is:

$$
=\frac{\text { Average Net Income }}{\text { Average Book Value }}=\frac{\$ 50,000}{\$ 250,000}
$$

"A project is acceptable if its average accounting return exceeds a target average accounting return"

- Drawbacks of AAR
- It is not a rate of return in any meaningful economic sense, as it ignores time value of money
- It lacks an objective or target AAR to be compared with
- It focuses on net income and book value rather than cash flow and market value
- Advantages
- Easy to calculate
- Needed information will usually be available


## Internal Rate of Return

- With IRR, we try to find a single rate of return that summarizes the merits of a project
- We want this rate to be an "internal" rate in the sense that it only depends on the cash flows of a particular investment, not on rates offered elsewhere
- Consider a project that costs $\$ 100$ today and pays $\$ 110$ in one year.
- Obviously it pays a return of $10 \%$, since it pays $\$ 1.10$ for every dollar we put in.
- This $10 \%$ is in fact the internal rate of return.


## "Based on IRR rule an investment is acceptable if the IRR exceeds the required return. It should be rejected otherwise"

- Now for our example, NPV for the investment at discount rate R is:

$$
\mathrm{NPV}=0=\frac{-\$ 100+\$ 110}{(1+I R R)}
$$

- Now if we don't know the discount rate, it represents a problem. But still we could ask, how high the discount rate would have to be before this project was unacceptable.
- The investment is economically a break-even proposition when the NPV is zero because the value is neither created nor destroyed.
- To find the break-even discount rate, we set NPV equal to zero and solve for R

$$
\begin{aligned}
& \mathrm{NPV}=0=\frac{-\$ 100+\$ 110}{(1+I R R)} \\
& 100=\frac{\$ 110}{(1+I R R)} \\
& (1+I R R)=\frac{\$ 110}{\$ 100}=1.10 \\
& \mathrm{R}=1.10-1 \\
& \mathrm{R}=10 \%
\end{aligned}
$$

- This $10 \%$ is what we already have called the return on this investment.
"The IRR on an investment is the required return that results in a zero NPV when it is used as the discount rate"
- The fact that the IRR is simply the discount rate that makes the NPV equal to zero is important because it tells us how to calculate the return on more complicated investments
- Suppose you are looking at an investment with the cash flows $\$ 60$ per year for two years
- With our understanding on calculating IRR by equating NPV to zero:

$$
\mathrm{NPV}=0=-100+\frac{60}{(1+I R R)}+\frac{60}{(1+I R R)^{2}}
$$

- The only way to find IRR in general is by trial and error method
- If we start with a $0 \%$ rate, then

$$
\text { NPV }=\$ 120-100=\$ 20
$$

- At $10 \%$ discount rate:

$$
\mathrm{NPV}=-100+\frac{60}{(1.10)}+\frac{60}{(1.10)^{2}}=\$ 4.13
$$

- We are getting closer!
- These and other possibilities are summarized in the following table:

| Discount Rate <br> $\mathbf{( \% )}$ | NPV <br> $\mathbf{( \$ )}$ |
| :---: | :---: |
| 0 | 20.00 |
| 5 | 11.56 |
| 10 | 4.13 |
| 15 | -2.46 |
| 20 | -8.33 |

Internal Rate of Return

- NPV appears to be zero between $10 \%$ and $15 \%$, so IRR is somewhere in that range
- With a little effort we can find that the IRR is about $13.1 \%$
- So if our required return is less than $13.1 \%$ we would take this investment, otherwise reject it.

- A project has total up-front cost of $\$ 435.44$. the cash flows are $\$ 100$ in 1 st year, $\$ 200$ in 2 nd year, and $\$ 300$ in 3 rd year
- What is the IRR?
- At $18 \%$ required return, should we take this investment?
- NPV calculated at different discount rates is as below:

| Discount Rate <br> $\mathbf{( \% )}$ | $\mathbf{N P V}$ <br> $\mathbf{( \$ )}$ |
| :---: | :---: |
| 0 | 164.56 |
| 5 | 100.36 |
| 10 | 46.15 |
| 15 | 0.00 |
| 20 | -39.61 |

- At $18 \%$ required rate, we should not take this investment
- IRR and NPV rules always lead to identical decisions as long as
- Project's cash flows are conventional i.e. first cash flow (initial investment) is negative and rest are positive.
- Projects are independent i.e. decision to accept or reject one project does not affect the decision to accept or reject any other.


## Problems with IRR

- Problems with IRR come about when
- cash flows are not conventional or
- when we are trying to compare two or more investments
- Non-conventional Cash Flows
- Suppose we have a mining project that requires a $\$ 60$ investment. Our cash flows in the 1 st year will be $\$ 155$. in the second year the mine is depleted, but we have to spend $\$ 100$ to restore the terrain
- Here both the 1 st and 3rd year cash flows are negative.


## Problems with IRR

- Non-conventional Cash Flows
- To find IRR, the NPV at different rates are

| Discount Rate <br> $\mathbf{( \% )}$ | NPV <br> $\mathbf{( \$ )}$ |
| :---: | :---: |
| 0 | -5.00 |
| 10 | -1.74 |
| 20 | -0.28 |
| 30 | 0.06 |
| 40 | -0.31 |



- The IRR of both $25 \%$ and $331 / 3 \%$ are correct, making it a Multiple rates of return problem
- In this example IRR rule breaks down completely
- Should we take this investment on $10 \%$ required rate? By IRR rule we should.
- But since NPV is negative at any discount rate less than $25 \%$, so this is not a good investment.
- NP is positive only if the required return is between $25 \%$ and $331 / 3 \%$
- Moral of the Story
- Cash flows are not conventional.
- So, the obvious question "What is the rate of return?" may not be answered, although NPV works just fine.
- Mutually Exclusive Investments
- If two investments X and Y are mutually exclusive, then taking one of them means that we can't take the other one.
- Two projects that are not mutually exclusive are said to be independent
- e.g. we can build a gas station OR an apartment building on a corner plot but not both
- So the question arises: "which investment is best?"
- Answer: The one with largest NPV
- Cash flows from two mutually exclusive events

| Year | Investment A <br> $\mathbf{( \$ )}$ | Investment B <br> $\mathbf{( \$ )}$ |
| :---: | :---: | :---: |
| $\mathbf{0}$ | 100 | 100 |
| $\mathbf{1}$ | 50 | 20 |
| $\mathbf{2}$ | 40 | 40 |
| $\mathbf{3}$ | 40 | 50 |
| $\mathbf{4}$ | 30 | 60 |

- IRR for A is $24 \%$ and IRR for B is $21 \%$
- NPV for the investments for different required returns

| Discount Rate <br> $\mathbf{( \% )}$ | NPV (A) <br> $\mathbf{( \$ )}$ | NPV (B) <br> $\mathbf{( \$ )}$ |
| :---: | :---: | :---: |
| 0 | 60.00 | 70.00 |
| 5 | 43.13 | 47.88 |
| 10 | 29.06 | 29.79 |
| 15 | 17.18 | 14.82 |
| 20 | 7.06 | 2.31 |
| 25 | -1.63 | -8.22 |

- IRR for A $(24 \%)$ is larger than that for B $(21 \%)$. However investment having higher NPV depends on required return.
- B has greater total cash flow but it pays back more slowly than A. So it has higher NPV at lower discount rates.
- If our required return is $10 \%$, B has higher NPV and is better of the two even though A has higher returns.
- If our required return is $15 \%$, then obviously A is better.

- At any discount rate less than $11.1 \%$, NPV for B is higher, so B benefits more than A , even though A's IRR is higher
- At any rate greater than $11.1 \%$, investment A has greater NPV
- Moral of the story
- Whenever we have mutually exclusive investments, we shouldn't rank them based on their returns
- In other words, IRR can be misleading in determining the best investment
- Instead we should look at their relative NPVs to avoid possibility of choosing incorrectly


## Qualities of IRR

- Most widely used
- Easily communicated and understood
- We can estimate IRR even if we don't know the discount rate


## PROFITABILITY INDEX

- Also defined as benefit cost ratio, this index is defined as the present value of the future cash flows divided by the initial investment.
- So if a project cost $\$ 200$ and the present value of its future cash flows is $\$ 220$, then the profitability index would be:

$$
\mathrm{PI}=\$ 220 / 200=1.10
$$

- Also, the NPV for this investment is $\$ 20$, so this a desirable investment.
- The profitability index for a positive NPV investment would be bigger than 1.00 and less than 1.00 for a negative NPV investment
- The PI of 1.10 tells us that per dollar invested, $\$ 1.10$ in value or $\$ 0.10$ in NPV results. So it measures value created per dollar invested.
- It is often proposed as a measure of performance of government or other non-profit investments.
- When capital is scarce, it is sensible approach to allocate it to those projects with highest PI
- Consider an investment which costs $\$ 5$ and has a $\$ 10$ present value and an investment costing $\$ 100$ with a $\$ 150$ present value.
- The first of these investments has an NPV of $\$ 5$ and a PI of 2, while the second one has an NPV of $\$ 50$ and a PI of 1.50 .
- If these are mutually exclusive investments, then the second one is preferred even though it has a lower PI
- This ranking problem is very similar to IRR ranking problem.
- Advantages
- Closely related to NPV, generally leading to identical decisions
- Easy to understand and communicate
- May be useful when available investment funds are limited
- Disadvantages
- May lead to incorrect decisions in comparisons of mutually exclusive investments


## Capital Budgeting Practice

- Why alternative procedures used if NPV seems to tell us directly what we want to know?
- We are trying to make an investment decision while operating under considerable uncertainty
- We can only estimate the NPV of an investment in this case but the true NPV might be quite different.
- Since the true NPV is unknown, the financial manager seeks clues to assess the reliability of the estimated NPV.
- For this purpose firms often use multiple criteria for evaluating a proposal.
- Suppose, we have an investment with positive NPV, with a short payback and a very high AAR.
- In this case all different indicators give a green signal. In other words, payback and AAR are consistent with the conclusion that NPV is positive.
- Now suppose, we have a positive estimated NPV, a long payback and a low AAR. This could be good investment but we should be much more careful in making the decision since we are getting conflicting signals.
- If the estimated NPV is based on projections in which we have little confidence then further analysis is probably in order.
- A Historical Comparison of Primary Use of Various Capital Budgeting Techniques

| Particular | $\mathbf{1 9 5 9}$ | $\mathbf{1 9 6 4}$ | $\mathbf{1 9 7 0}$ | $\mathbf{1 9 7 5}$ | $\mathbf{1 9 7 7}$ | $\mathbf{1 9 7 9}$ | $\mathbf{1 9 8 1}$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Payback Period | $34 \%$ | $24 \%$ | $12 \%$ | $15 \%$ | $9 \%$ | $10 \%$ | $5 \%$ |
| AAR | 34 | 30 | 26 | 10 | 25 | 14 | 10.7 |
| IRR | 19 | 38 | 57 | 37 | 54 | 60 | 65.3 |
| NPV | -- | --- | -- | 26 | 10 | 14 | 16.5 |
| IRR or NPV | 19 | 38 | 57 | 63 | 64 | 74 | 81.8 |

- Percentage of CFOs who always Use a Given Technique in 2000

| Capital Budgeting Technique | Percentage Always or <br> Almost Always Use | Overall | Average Score <br> Large <br> Firms | Small <br> Firms |
| :--- | :---: | :---: | :---: | :---: |
| Internal Rate of Return | $76 \%$ | 3.09 | 3.41 | 2.87 |
| Net Present Value | 75 | 3.08 | 3.42 | 2.83 |
| Payback Period | 57 | 2.53 | 2.55 | 2.72 |
| Discounted payback period | 29 | 1.56 | 1.55 | 1.58 |
| Average Accounting Return | 20 | 1.34 | 1.25 | 1.41 |
| Profitability Index | 12 | 0.83 | 0.75 | 0.88 |

## Summary of Investment Criteria

## Discounted Cash Flow Criteria

- Net Present Value (NPV)
- The NPV of an investment is the difference between its market value and its cost.
- The NPV rule is to take a project if its NPV is positive.
- It is estimated by calculating the present value of the future cash flows and then subtracting the cost.
- Internal Rate of Return
- IRR is the discount rate that makes the estimated NPV of an investment equal to zero
- IRR rule is to take a project when its IRR exceeds the required return.
- It leads to the same decision as NPV for conventional, independent projects.
- For mutually exclusive and non-conventional projects, IRR may be misleading.
- Profitability Index
- PI is the ratio of present value to cost. It measures the present value of an investment per dollar invested.
- PI rule is to take an investment if the index exceeds 1.
- It is sometimes use to rank projects when a firm has more positive NPV investments than it can currently finance.


## Accounting Criteria

- Average Accounting Return
- AAR is a measure of accounting profit relative to book value.
- AAR rule is to take an investment if its AAR exceeds a benchmark AAR.


## Payback Criteria

- Payback Period
- It is the length of time until the sum of an investment's cash flows equals its costs.
- Payback period rule is to take a project if its payback is less than some cutoff.
- It ignores risk, time value of money and cash flows beyond cutoff point.


## MAKING CAPITAL INVESTMENT DECISIONS

- We shall now spread the numbers for a proposed investment or project and, based on those numbers make an initial assessment about whether or not the project should be undertaken.
- We will focus on the process of setting up a discounted cash flow analysis.
- While evaluating a proposed investment, special attention is paid to the relevance of the information available for decision making.


## Project Cash Flows

- To evaluate a proposed investment, we must consider the changes in the firm's cash flows and then decide whether or not they add value to the firm.
- The first step is, thus, to decide which cash flows are relevant and which are not.


## Relevant Cash Flows

- A relevant cash flow for a project is a change in the firm's overall future cash flow that comes about as direct consequence of the decision to take the investment.
- Since the relevant cash flows are defined in terms of changes in, or increments to, the firm's existing cash flow, they are called the incremental cash flows associated with the project.


## Incremental Cash Flows

- The incremental cash flows for project evaluation consist of any and all changes in the firm's future cash flows that are a direct consequence of taking the project.


## The Stand-Alone Principle

- The assumption that evaluation of a project may be based on the project's incremental cash flows
- Analysis of the project as a "minifirm" with its own future revenues and costs, its own assets and its own cash flows


## Incremental Cash Flows

- It seems easy enough to decide whether a cash flow is incremental or not. Even so there are situations where mistakes are easy to make.
- Sunk Costs
- Opportunity Costs
- Side Effects
- Net Working Capital
- Financing Costs
- Other Issues
- Sunk Costs
- A cost that has already been incurred and cannot be recouped and therefore should not be considered in an investment decision.
- e.g. a consultant's fee for evaluating the option of launching a new product.
- Opportunity Costs
- The most valuable alternative that is given up if a particular investment is undertaken
- e.g. option of selling a piece of land (bought years ago) at market rates instead of establishing a school upon it
- How much should be charged to school project in terms of opportunity costs; amount at which we bought the piece of land or its current market price?
- Side Effects
- It is not unusual for a project to have side or spillover effects both good and bad.
- e.g. sales of a new car by a certain company might come at the expense of the other cars of the same company.
- This phenomenon is called erosion, piracy or cannibalism.
- So cash flows from the new product line should be adjusted downward to reflect lost profits on other lines.
- Net Working Capital
- Normally a project requires investments in net working capital in addition to long term assets.
- e.g. cash in hand, inventories \& accounts receivables as well as accounts payables; the balance being the net working capital.
- As the project winds down, net working capital gets freed-up. So investment in NWC resembles a loan as firm provides NWC at the beginning and recovers it towards the end.
- Financing Costs
- We don't include interest paid or any other financing costs like dividend or principle paid while analyzing a proposed investment, as we are interested in the cash flow generated by the assets of the project.
- The mixture of debt and equity a firm actually chooses to use in financing a project is a managerial variable and does not form part of the project evaluation process.
- Other Issues
- We are only concerned in measuring cash flows and when it actually occurs rather than when it occurs in accounting sense.
- We are interested in after-tax cash flows as in fact incremental cash flows means after-tax incremental cash flows.


## Pro Forma Financial Statements

- Pro forma financial statements project future years' operations in a summarized format.
- To prepare these statements, we need estimates of quantities like
- Unit sales
- Selling price per unit
- Variable cost per unit
- Total fixed cost
- Suppose we want to prepare a set of pro forma financial statements for a project for ND Enterprises. In order to do so, we must have some background information. In this case, assume:
- Sales of 10,000 units/year @ \$5/unit
- Variable cost/unit is $\$ 3$. Fixed costs are $\$ 5,000 /$ year. Project has no salvage value. Project life is 3 years.
- Project cost is $\$ 21,000$. Depreciation is $\$ 7,000 /$ year.
- Investment in net working capital is $\$ 10,000$.
- The firm's required return is $20 \%$. The tax rate is $34 \%$.


## Projected Income Statements

| Particular | Amount |
| :--- | ---: |
| Sales (10,000 units / year @ \$5 per unit) | $\$ 50,000$ |
| Variable Cost (\$3 / Unit) | 30,000 |
| Gross Profit | $\$ 20,000$ |
| Fixed Cost | 5,000 |
| Depreciation | 7,000 |
| EBIT | $\$ 8,000$ |
| Taxes (34 \%) | 2,720 |
| Net Income | $\$ 5,280$ |

## Projected Balance Sheets

| Particular | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ |
| :--- | :---: | ---: | ---: | ---: |
| Net working Capital | $\$ 10,000$ | $\$ 10,000$ | $\$ 10,000$ | $\$ 10,000$ |
| Net Fixed Assets | 21,000 | 14,000 | 7,000 | 0 |
| Total | $\$ 31,000$ | $\$ 24,000$ | $\$ 17,000$ | $\$ 10,000$ |

## Projected Cash Flows

We know that

Project Cash Flow $=$ Project operating cash flow - Project change in net working capital - Project capital spending

We also know that
Operating Cash flows $=$ Earnings before interest and taxes + Depreciation - Taxes
Let's use the information from the previous example to do a capital budgeting analysis.

Projected Operating Cash Flows (OCF):

| Particular | Amount |
| :--- | :---: |
| EBIT | $\$ 8,000$ |
| Depreciation | $+7,000$ |
| Taxes | $-2,720$ |
| Operating Cash Flow | $\$ 12,280$ |
|  |  |

Projected Cash Flows

| Particular | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ |
| :--- | ---: | ---: | ---: | ---: |
| Operating Cash Flow | --- | $\$ 12,280$ | $\$ 12,280$ | $\$ 12,280$ |
| Changes in NWC | $-10,000$ | 0 | 0 | 10,000 |
| Capital Spending | $-21,000$ | 0 | 0 | 0 |
| Total | $-31,000$ | 12,280 | 12,280 | 22,280 |

- Now we know that firm must spend $\$ 21,000$ up front for fixed assets and invest an additional $\$ 10,000$ in net working capital. So the immediate outflow is $\$ 31,000$
- The recovery of $\$ 10,000$ tied up in NWC in the last year will lead to a cash inflow of the same amount.

Capital Budgeting Evaluation:

$$
\begin{aligned}
& \mathrm{NPV}=-31,000+\frac{\$ 12,280}{(1.20)}+\frac{\$ 12,280}{(1.20)^{2}}+\frac{\$ 22,280}{(1.20)^{3}} \\
& \mathrm{NPV}=\$ 655 \\
& \mathrm{IRR}=21 \% \\
& \text { Payback }=2.3 \text { Years } \\
& \mathrm{AAR}=\$ 5280 /\{(31,000+24,000+17,000+10,000) / 4\} \\
& \mathrm{AAR}=25.76 \%
\end{aligned}
$$

- Should the firm invest in this project? Why or why not?


## PRO FORMA FINANCIAL STATEMENTS

Suppose we want to prepare a set of pro forma financial statements for a project for ND Enterprises. In order to do so, we must have some background information. In this case, assume:

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## Projected Income Statements

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| :--- | ---: |
| Sales (10,000 units / year @ $\$ 5$ per unit) | $\$ 50,000$ |
| Variable Cost (\$3 / Unit) | 30,000 |
| Gross Profit | $\$ 20,000$ |
| Fixed Cost | 5,000 |
| Depreciation | 7,000 |
| EBIT | $\$ 8,000$ |
| Taxes (34 \%) | 2,720 |
| Net Income | $\underline{\$ 5,280}$ |

## Projected Balance Sheets

| Particular | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ |
| :--- | ---: | ---: | ---: | ---: |
| Net working Capital | $\$ 10,000$ | $\$ 10,000$ | $\$ 10,000$ | $\$ 10,000$ |
| Net Fixed Assets | 21,000 | 14,000 | 7,000 | 0 |
| Total | $\$ 31,000$ | $\$ 24,000$ | $\$ 17,000$ | $\$ 10,000$ |

## Projected Cash Flows

We know that

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| Taxes | $-2,720$ |
| Operating Cash Flow | $\$ 12,280$ |
|  |  |

## Projected Cash Flows

| Particular | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{c}$ | $\mathbf{2}$ |
| :--- | ---: | ---: | ---: | ---: |

- Now we know that firm must spend $\$ 21,000$ up front for fixed assets and invest an additional $\$ 10,000$ in net working capital. So the immediate outflow is $\$ 31,000$
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& \mathrm{NPV}=\$ 655 \\
& \mathrm{IRR}=21 \% \\
& \text { Payback }=2.3 \text { Years } \\
& \mathrm{AAR}=\$ 5280 /\{(31,000+24,000+17,000+10,000) / 4\} \\
& \mathrm{AAR}=25.76 \%
\end{aligned}
$$

Should the firm invest in this project? Why or why not?

## The Tax Shield Approach

The tax shield definition of operating cash flow is:

$$
\begin{aligned}
& \text { OCF }=(\text { Sales }- \text { Costs }) \times(1-T)+\text { Dep. } \mathrm{X} T \\
& \text { Where T is the corporate tax rate }(34 \% \text { in this case }) \\
& \mathrm{OCF}=(\$ 50,000-35,000) \times 0.66+7,000 \times 0.34 \\
& \mathrm{OCF}=\$ 9,900+2,380=\$ 12,280
\end{aligned}
$$

This shows that operating cash flows has two components:

- Project's would be cash flows, had there been no depreciation expense i.e. \$9,900
- Depreciation tax shield $=\$ 2,380$


## A Closer Look on NWC

- Until now we have not explicitly considered the facts that
- Some of the sales might be on credit, and
- We may not have paid all the costs shown
- These are not big problems until we don't forget to include additions to net working capital in our analysis.
- Suppose we had the following income statement during a year of a project.
- 

| Sales | $\$ 500$ |
| :--- | ---: |
| Costs | $\underline{310}$ |
| Net income | $\underline{\$ 190}$ |

- We assume here that
- Depreciation and taxes are zero,
- No fixed assets are purchased during the year
- Only components of NWC are accounts receivable and accounts payable

| Particular | Beginning of Year | End of Year | Change |
| :--- | :---: | :---: | :---: |
| Accounts Receivable | $\$ 880$ | $\$ 910$ | $+\$ 30$ |
| Accounts Payable | 550 | 605 | +55 |
| Net Working Capital | $\$ 330$ | $\$ 305$ | -25 |
|  |  |  |  |

- Operating cash flow in this example is the same as EBIT as there are not taxes or depreciation
- NWC declined by $\$ 25$, there was no capital spending during the year. So,

```
Total cash flow \(=\) Operating cash flow - change in NWC - Capital spending
Total cash flow \(=\$ 190-(-25)-0\)
Total cash flow \(=\$ 215\)
```

- Now we need to know
- What were cash revenues for the year?
- What were cash costs?
- We had sales of $\$ 500$, while accounts receivable rose by $\$ 30$ over the year, so cash inflow is $\$ 500-30=\$ 470$
- We show costs of $\$ 310$ on our income statement but accounts payable increase by $\$ 55$ during the year. So, the cash costs for the year are $\$ 310-55=\$ 255$
- Cash inflows less cash outflows is $\$ 470-255=\$ 215$ (matching our previous result)

$$
\begin{aligned}
\text { Cash Flow } & =\text { cash inflow }- \text { cash outflow } \\
& =(\$ 500-30)-(310-55) \\
& =(\$ 500-310)-(30-55) \\
& =\mathrm{OCF}-\text { Change in NWC } \\
& =\$ 190-(-25) \\
& =\$ 215
\end{aligned}
$$

## The Pencil Company

- For the year just completed, sales were $\$ 998$ and costs $\$ 734$. following are the beginning and ending balance sheets

| Particular | Beginning of Year | End of Year |
| :--- | :---: | :---: |
| Accounts Receivable | $\$ 100$ | $\$ 110$ |
| Inventory | 100 | 80 |
| Accounts Payable | 100 | 70 |
| Net Working Capital | $\$ 100$ | $\$ 120$ |

- Sales were $\$ 998$ but receivables rose by only $\$ 10$. So cash revenues were $\$ 998-10=\$ 988$
- Costs were $\$ 734$ but inventories fell by $\$ 20$ and payables by $\$ 30$. Thus cash costs were:

$$
\$ 734-20+30=\$ 744
$$

- Net cash flow is:

$$
\$ 988-744=\$ 244
$$

- Accounting depreciation is a non-cash deduction. As a result depreciation has cash flow consequences only because it influences the tax bill.
- So, the way the depreciation is computed becomes relevant for capital investment decisions.
- Modified ACRS Depreciation
- The basic idea is that every asset is assigned to a particular class. The class establishes asset's life for tax purposes
- We compute the depreciation for each year by multiplying the cost of the asset by a fixed percentage.
- The expected salvage value and the actual expected economic life are not explicitly considered in calculation of depreciation
- MACRS Property Classes
Class Examples

| 3 -year | Equipment used in research |
| :--- | :--- |
| 5 -year | Autos, Computers |
| 7 -year | Most of industrial equipment |


| Year | Property Classes |  |  |
| :---: | :---: | :---: | :---: |
|  | 3-Year | 5-Year | 7-Year |
| 1 | $33.33 \%$ | $20.00 \%$ | $14.29 \%$ |
| 2 | 44.44 | 32.00 | 24.49 |
| 3 | 14.82 | 19.20 | 17.49 |
| 4 | 7.41 | 11.52 | 12.49 |
| 5 | - | 11.52 | 8.93 |
| 6 | - | 5.76 | 8.93 |
| 7 | - | - | 8.93 |
| 8 | - | - | 4.45 |

- Depreciation for an automobile costing $\$ 12,000$

| Year | MACRS \% | Depreciation |
| :---: | :---: | :---: |
| 1 | $20.00 \%$ | $\$ 2,400.00$ |
| 2 | 32.00 | $3,840.00$ |
| 3 | 19.20 | $2,304.00$ |
| 4 | 11.52 | $1,382.40$ |
| 5 | 11.52 | $1,382.40$ |
| 6 | 5.76 | 691.20 |
|  | $100.00 \%$ | $\$ 12,000.00$ |

- Book Value versus Market Value

| Year | Beginning Book Value | Depreciation | Ending Book Value |
| :---: | :---: | :---: | :---: |
| 1 | $\$ 12,000.00$ | $\$ 2,400.00$ | $\$ 9,600.00$ |
| 2 | $9,600.00$ | $3,840.00$ | $5,760.00$ |
| 3 | $5,760.00$ | $2,304.00$ | $3,456.00$ |
| 4 | $3,456.00$ | $1,382.40$ | $2,073.60$ |
| 5 | $2,073.60$ | $1,382.40$ | 691.20 |
| 6 | 691.20 | 691.20 | 0.00 |

DEPRECIATION

- Depreciation for an automobile costing $\$ 12,000$

| Year | MACRS \% | Depreciation |
| :---: | :---: | :---: |
| 1 | $20.00 \%$ | $\$ 2,400.00$ |
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| 3 | 19.20 | $2,304.00$ |
| 4 | 11.52 | $1,382.40$ |
| 5 | 11.52 | $1,382.40$ |
| 6 | 5.76 | 691.20 |
|  | $100.00 \%$ | $\$ 12,000.00$ |

- Book Value versus Market Value

| Year | Beginning Book Value | Depreciation | Ending Book Value |
| :---: | :---: | :---: | :---: |
| 1 | $\$ 12,000.00$ | $\$ 2,400.00$ | $\$ 9,600.00$ |
| 2 | $9,600.00$ | $3,840.00$ | $5,760.00$ |
| 3 | $5,760.00$ | $2,304.00$ | $3,456.00$ |
| 4 | $3,456.00$ | $1,382.40$ | $2,073.60$ |
| 5 | $2,073.60$ | $1,382.40$ | 691.20 |
| 6 | 691.20 | 691.20 | 0.00 |

- Suppose we want to sell the car after 5 years. Based on historical averages, it will be worth, say $25 \%$ of the purchase price, i.e. $0.25 \times 12,000=\$ 3,000$
- -If sold on this price we would have to pay taxes on the difference between the price of $\$ 3,000$ and the book value of $\$ 691.20$.
- So, the tax liability would be

$$
0.34 \$ 2,308.80=\$ 784.99
$$

- The reason for tax payment is that the difference between the market and book value is the excess depreciation that must be recaptured when the asset is sold.
- This is not a tax on capital gain, because a capital gain occurs if the market price exceeds the original cost.
- If the book value exceeds the market value, then the difference is treated as a loss for tax purposes.


## The SS Company

- The company just purchased a new IT system with an installed cost of $\$ 160,000$. the computer is treated as 5 -year property under MACRS.
- What are the yearly depreciation allowances?
- The system will have an estimated worth of only $\$ 10,000$, if sold in 4 years.
- What are the tax consequences of the sale?
- What is the total after tax cash flow from sale?

| Year | MACRS \% | Depreciation | Ending Book Value |
| :---: | :---: | :---: | :---: |
| 1 | $20.00 \%$ | $\$ 32,000$ | $\$ 128,000$ |
| 2 | 32.00 | 51,200 | 76,800 |
| 3 | 19.20 | 30,720 | 46,080 |
| 4 | 11.52 | 18,432 | 27,648 |
| 5 | 11.52 | 18,432 | 9,216 |
| 6 | 5.76 | 9,216 | 0 |
|  | $100.00 \%$ | $\$ 160,000$ | - |

- The book value at the end of year 4 is $\$ 27,648$. if we sell the system for $\$ 10,000$ at that time, we will have a loss of $\$ 17,648$ for tax purposes.
- We get $\$ 10,000$ from buyer.
- We save $0.34 \times \$ 17,648=\$ 6,000$ in taxes. So the total after tax cash flow from the sale is a $\$ 16,000$ cash inflow.


## The M Inc.

- The company is considering the feasibility of a new line of product. Based on the market research, it projects unit sales as given:

| Year | Unit Sales |
| :---: | :---: |
| 1 | 3,000 |
| 2 | 5,000 |
| 3 | 6,000 |
| 4 | 6,500 |
| 5 | 6,000 |
| 6 | 5,000 |
| 7 | 4,000 |
| 8 | 3,000 |

- The new product will be priced to sell at $\$ 120$ per unit to start. However the competitions may affect the price to drop to $\$ 110$ after three years.
- The project will require $\$ 20,000$ in net working capital at the start. Subsequently, total NWC at the end of each year will be $15 \%$ of sales for that year.
- The variable cost per unit is $\$ 60$ and total fixed costs are $\$ 25,000$ per year.
- It will cost about $\$ 800,000$ to buy the equipment necessary to begin the production. This equipment comes under 7 -year MACRS property.
- The equipment will be worth $20 \%$ of its cost in 8 years, or $0.20 \times \$ 800,000=\$ 160,000$. Tax rate is $34 \%$ and required return is $15 \%$
- Should the company proceed?

| Year | Unit Price | Unit Sale | Revenues |
| :---: | :---: | :---: | :---: |
| 1 | $\$ 120$ | 3,000 | $\$ 360,000$ |
| 2 | 120 | 5,000 | 600,000 |
| 3 | 120 | 6,000 | 720,000 |
| 4 | 110 | 6,500 | 715,000 |
| 5 | 110 | 6,000 | 660,000 |
| 6 | 110 | 5,000 | 550,000 |
| 7 | 110 | 4,000 | 440,000 |
| 8 | 110 | 3,000 | 330,000 |


| Year | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ |
| :--- | ---: | ---: | ---: | ---: |
| Unit Price | $\$ 120$ | $\$ 120$ | $\$ 120$ | $\$ 110$ |
| Unit Sale | 3,000 | 5,000 | 6,000 | 6,500 |
| Revenues | $\$ 360,000$ | $\$ 600,000$ | $\$ 720,000$ | $\$ 715,000$ |
| Variable Cost | 180,000 | 300,000 | 360,000 | 390,000 |
| Fixed Cost | 25,000 | 25,000 | 25,000 | 25,000 |
| Depreciation | 114,320 | 195,920 | 139,920 | 99,920 |
| EBIT | $\$ 40,680$ | $\$ 79,080$ | $\$ 195,080$ | $\$ 200,080$ |
| Taxes $(34 \%)$ | 13,381 | 26,887 | 66,327 | 68,027 |
| Net Income | $\$ 26,849$ | $\$ 52,193$ | $\$ 128,573$ | $\$ 132,053$ |


| Year | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |
| :--- | ---: | ---: | ---: | ---: |
| Unit Price | $\$ 110$ | $\$ 110$ | $\$ 110$ | $\$ 110$ |
| Unit Sale | 6,000 | 5,000 | 4,000 | 3,000 |
| Revenues | $\$ 660,000$ | $\$ 550,000$ | $\$ 440,000$ | $\$ 330,000$ |
| Variable Cost | 360,000 | 300,000 | 240,000 | 180,000 |
| Fixed Cost | 25,000 | 25,000 | 25,000 | 25,000 |
| Depreciation | 71,440 | 71,440 | 71,440 | 35,600 |
| EBIT | $\$ 203,560$ | $\$ 153,560$ | $\$ 103,560$ | $\$ 89,400$ |
| Taxes $(34 \%)$ | 69,210 | 52,210 | 35,210 | 30,396 |
| Net Income | $\$ 134,350$ | $\$ 101,350$ | $\$ 68,350$ | $\$ 59,004$ |

## Changes in Net Working Capital

- Net working Capital starts out at $\$ 20,000$ and then rises to $15 \%$ of sales.
- For first year, net working capital grows from $\$ 20,000$ to $0.15 \times 360,000=\$ 54,000$
- The increase is thus $\$ 54,000-20,000=\$ 34,000$
- Rest of the figures are given in the next table

| Year | Revenue | Net Working Capital | Cash Flow |
| :---: | :---: | :---: | :---: |
| 0 | - | $\$ 20,000$ | $-\$ 20,000$ |
| 1 | $\$ 360,000$ | 54,000 | $-34,000$ |
| 2 | 600,000 | 90,000 | $-36,000$ |
| 3 | 720,000 | 108,000 | $-18,000$ |
| 4 | 715,000 | 107,250 | 750 |
| 5 | 660,000 | 99,000 | 8,250 |
| 6 | 550,000 | 82,500 | 16,500 |
| 7 | 440,000 | 66,000 | 16,500 |
| 8 | 330,000 | 49,500 | 16,500 |

## Capital Spending

- Invested $\$ 800,000$ at year 0 , which by assumption will be worth $\$ 160,000$ at the end of the project. It will have a book value of zero at that time.
- This $\$ 160,000$ excess of market value over book value is taxable. So after tax proceeds will be:

$$
\$ 160,000 \times(1-0.34)=\$ 105,600
$$

- Project Total Cash Flows

| Year | $\mathbf{0}$ | $\mathbf{1}$ | $\mathbf{2}$ | $\mathbf{3}$ | $\mathbf{4}$ |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Operating CF |  | $\$ 141,169$ | $\$ 248,113$ | $\$ 268,113$ | $\$ 231,973$ |
| NWC Changes | $-20,000$ | $-34,000$ | $-36,000$ | $-18,000$ | 750 |
| Capital Spending | $-800,000$ |  |  |  |  |
| Total Project Cash Flow | $-\$ 820,000$ | $\$ 107,169$ | $\$ 212,113$ | 250,673 | 232,723 |
| Cumulative Cash Flow | $-\$ 820,000$ | $-\$ 712,831$ | $-\$ 500,718$ | $-\$ 250,045$ | $-\$ 17,322$ |
| Discounted Cash flow @15\% | $-820,000$ | 93,190 | 160,388 | 164,821 | 133,060 |


| Year | $\mathbf{5}$ | $\mathbf{6}$ | $\mathbf{7}$ | $\mathbf{8}$ |
| :--- | ---: | ---: | ---: | ---: |
| Operating CF | $\$ 205,790$ | $\$ 172,790$ | $\$ 139,790$ | $\$ 94,064$ |
| NWC Changes | 8,250 | 16,500 | 16,500 | 66,000 |
| Capital Spending |  |  |  | $\$ 105,600$ |
| Total Project Cash Flow | $\$ 214,040$ | $\$ 189,290$ | $\$ 156,290$ | $\$ 266,204$ |
| Cumulative Cash Flow | 196,718 | $\$ 386,008$ | $\$ 542,298$ | $\$ 808,502$ |
| Discounted Cash flow @15\% | 106,416 | 81,835 | 58,755 | 87,023 |

- Net Present Value at $15 \%$ is calculated to be $\$ 65,488$, indicating that the project is acceptable
- Internal rate of return is greater than $15 \%$ since NPV is positive, and is calculated to be $\$ 17.24 \%$, again indicating the acceptance of the project
- Payback period is calculated to be 4.08 years.


## RETURNS

- One of the responsibilities of the financial manager is to assess the value of the proposed investment.
- In doing this, it is important that we first look at what financial investments have to offer.
- At a minimum, the return we require from a proposed non-financial investment must be at least as large as what we can get from buying financial assets of similar risk.
- Lessons from market history:
- There is a reward for bearing risk
- The greater the potential reward, the greater the risk.
- If you buy an asset of any sort, your gain (or loss) from that investment is called your return on investment.
- This return, usually termed as dollar returns, has normally two components:
- Income earned (Dividend)
- Capital gain
- Alternatively, dollar returns are the sum of the cash received and the change in dollar value of the asset.

- Suppose you bought 100 shares of a corporation one year ago at $\$ 25$.
- Over the last year, you received $\$ 20$ ( $=20$ cents per share $\times 100$ shares) in dividends.
- At the end of the year, the stock sells for $\$ 30$.
- How did you do?
- You invested $\$ 25 \times 100=\$ 2,500$.
- At the end of the year, you have stock worth $\$ 3,000$ and cash dividends of $\$ 20$.
- Your dollar gain was $\$ 520=\$ 20+(\$ 3,000-\$ 2,500)$.
- Dollar returns: $\$ 520$ gain

- If you sell the stock at the end of the year:

$$
\begin{aligned}
& \text { Total Cash Inflow }=\$ 2,500+520 \\
& \text { Total Cash Inflow }=\$ 3,020
\end{aligned}
$$

- If you don't sell the stocks of the company, rather hold it
- You should still consider the capital gain as part of your return
- It is not just a paper gain


## Percentage Returns

- It is more convenient to summarize the information about returns in percentage terms, as in this way, your return does not depend on how much you actually invest.
- Dividend Yield = Dividend / beginning price
- Capital Gains Yield = (ending price - beginning price) $/$ beginning price
- Total Percentage return $=$ dividend yield + capital gains yield

$$
\begin{aligned}
\text { Percentage Return } & =\frac{\text { Dollar Return }}{\text { Beginning Market Value }} \\
& =\frac{\text { Dividend }+ \text { Change in Market Value }}{\text { Beginning Market Value }} \\
& =\text { Dividend Yield }+ \text { Capital Gains Yield }
\end{aligned}
$$

- In our example:
- Dividend Yield $=\$ 20 / 2,500=0.8 \%$
- Capital Gains Yield $=(\$ 3,000-2,500) / 2,500=20 \%$
- Total Percentage Return $=20 \%+0.8 \%=20.8 \%$

- Suppose you buy some stock for $\$ 25$ per share. At the end of the year, the price is $\$ 35$ per share. During the year, you get a $\$ 2$ dividend per share
- What is the Dividend Yield? Capital Gains Yield? Total Percentage Return?
- If your total investment was $\$ 1,000$ how much do you have at the end of the year?


## Inflows

## Outflows



- Dividend yield $=\$ 2 / 25=0.08=8 \%$
- Capital gains yield $=(\$ 35-25) / 25=40 \%$
- Percentage Return $=8 \%+40 \%=48 \%$
- So, if you had invested $\$ 1,000$, you would have had $\$ 1,480(1,000 \times 48 \%)$ at the end of the year.


## Variability of Returns

- We know that year-to-year returns on common stocks tend to be more volatile than the returns on long term bonds.
- Measuring this variability is the key factor in examining the topic of risk.
- First we draw a frequency distribution for the common stock returns to count up the number of times the annual return on large stock portfolios falls within a certain percentage range.

Frequency Distribution of Returns on Common Stocks for 72 years


- Now we need to measure the spread in returns.
- For example, if the returns on small stock in a particular year was $17.3 \%$, we need to know how far the actual return deviate from this average, or we need a measure of the volatility of returns.
- The most commonly used measures for this purpose are variance and its square root, the standard deviation.
- Variance measures the average squared differences between the actual returns and the average return. The biggest the number is the more the actual returns tend to differ from the average return.
- The larger the variance or standard deviation is the more spread out the returns will be.
- Suppose, a particular investment had returns of $10 \%, 12 \%, 3 \%$ and $-9 \%$ over the last four years.
- Average return is $(0.10+0.12+0.03-0.09) / 4=4 \%$
- To compute the variance, we take deviations of the actual returns from this average return, take squares of these deviations and the average of the squared deviations will be our variance.

| Year | Actual <br> Return | Average <br> Return | Deviation from <br> the Mean | Squared <br> Deviation |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 0.10 | 0.04 | 0.06 | 0.0036 |
| 2 | 0.12 | 0.04 | 0.08 | 0.0064 |
| 3 | 0.03 | 0.04 | -0.01 | 0.0001 |
| 4 | $\underline{-0.09}$ | 0.04 | $\underline{-0.13}$ | $\underline{0.0169}$ |
| Total | 0.16 |  | 0.00 | 0.0270 |

Variance $=0.0270 /(4-1)=0.009$
Standard Deviation $=\sqrt{ } 0.009=.09487$ or $9.487 \%$

- In general, the variance for $T$ historical returns is:

$$
\operatorname{Var}(\mathrm{R})=\frac{\left.\left.\left(R_{1}-R\right)^{2}\right)+\ldots \ldots \ldots \ldots \ldots .+\left(R_{T}-R\right)^{2}\right)}{(T-1)}
$$

- The standard deviation is always the square root of the variance.

VARIABILITY OF RETURNS

- In general, the variance for T historical returns is:

$$
\operatorname{Var}(\mathrm{R})=\frac{\left.\left.\left(R_{1}-R\right)^{2}\right)+\ldots \ldots \ldots \ldots \ldots .+\left(R_{T}-R\right)^{2}\right)}{(T-1)}
$$

- The standard deviation is always the square root of the variance.

| Year | Company X Return | Company Y Return |
| :---: | :---: | :---: |
| 2000 | -0.20 | 0.05 |
| 2001 | 0.50 | 0.09 |
| 2002 | 0.30 | -0.12 |
| 2003 | 0.10 | 0.20 |

- What are the average returns, variances, standard deviations?
- Which investment was more volatile?
- Company X average returns,

$$
\mathrm{R}_{\mathrm{x}}=(-.20+.50+.30+.10) / 4=.175
$$

- Company Y average returns,

$$
\mathrm{R}_{\mathrm{Y}}=(.05+.09-.12+.20) / 4=.055
$$

- Now, we calculate variance for Company X

| Year | Actual <br> Return | Average <br> Return | Deviation from <br> the Mean | Squared <br> Deviation |
| :---: | :---: | :---: | :---: | :---: |
| 2000 | -0.20 | 0.175 | -0.375 | 0.140625 |
| 2001 | 0.50 | 0.175 | 0.325 | 0.105625 |
| 2002 | 0.30 | 0.175 | 0.125 | 0.015625 |
| 2003 | $\underline{0.10}$ | 0.175 | $\underline{-0.075}$ | $\underline{0.005623}$ |
| Total | 0.70 |  | 0.000 | 0.267500 |

- Summarizing the calculations:

|  | Company X | Company Y |
| :--- | :--- | :--- |
| Variance $\left(\sigma^{2}\right)$ | $0.2675 / 3=0.0892$ | $0.0529 / 3=0.0176$ |
| Standard Deviation $(\sigma)$ | $\sqrt{ } 0.0892=0.2987$ | $\sqrt{ } 0.0176=0.1327$ |

- Standard deviation for company X, $29.87 \%$ more than twice Company Y's $13.27 \%$. This indicates that company X is more volatile investment.


## Expected Return

- Consider a single period of time, say a year. We have two stocks, L and U. and they are expected to have a return of $25 \%$ and $20 \%$ respectively in the coming year
- Why to invest and hold stock U?
- Stock L having higher expected return may actually perform abnormally. It could go up to $70 \%$ in economic boom or may slide to $-20 \%$ in recession.
- While stock U may earn $30 \%$ in recession and $10 \%$ during a boom.
- If the probabilities of occurring a boom and a recession are 0.5 each then the expected return for both the stocks can be calculated as:

| State of <br> Economy | $\mathbf{P}$ | Stock $\mathbf{L}$ |  | Stock $\mathbf{U}$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | $\mathbf{R}_{\mathbf{L}}$ | $\mathbf{P} \times \mathbf{R}_{\mathbf{L}}$ | $\mathbf{R}_{\mathbf{U}}$ | $\mathbf{P} \times \mathbf{R}_{\mathbf{U}}$ |  |
| Recession | 0.50 | -0.20 | -0.10 | 0.30 | 0.15 |
| Boom | $\underline{0.50}$ | 0.70 | $\underline{0.35}$ | 0.10 | $\underline{0.05}$ |
|  | 1.00 | $\mathrm{E}\left(\mathrm{R}_{\mathrm{L}}\right)$ | 0.25 | $\mathrm{E}\left(\mathrm{R}_{\mathrm{L}}\right)$ | 0.20 |

## Risk Premium

- The difference between the return on a risky investment and that on a risk-free investment.
- Suppose, risk-free investments are currently offering 8 percent. In other words, risk-free rate $\left(R_{f}\right)$ is $8 \%$.
- Using this information, we can calculate the projected risk premia on stocks $U$ and $L$.

$$
\begin{aligned}
\text { Risk Premium } & =\text { expected return }- \text { Risk-free rate } \\
\text { Risk Premium (U) } & =\mathrm{E}\left(\mathrm{R}_{\mathrm{U}}\right)-\mathrm{R}_{\mathrm{f}} \\
& =20 \%-8 \% \\
& =12 \% \\
\text { Risk Premium (L) } & =\mathrm{E}\left(\mathrm{R}_{\mathrm{L}}\right)-\mathrm{R}_{\mathrm{f}} \\
& =25 \%-8 \% \\
& =17 \%
\end{aligned}
$$

- Unequal Probabilities Case:

| State of | $\mathbf{P}$ | Stock $\mathbf{L}$ |  | Stock $\mathbf{U}$ |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Economy | $\mathbf{R}_{\mathbf{L}}$ | $\mathbf{P} \times \mathbf{R}_{\mathbf{L}}$ | $\mathbf{R}_{\mathbf{U}}$ | $\mathbf{P} \times \mathbf{R}_{\mathbf{U}}$ |  |
| Recession | 0.80 | -0.20 | -0.16 | 0.30 | 0.24 |
| Boom | $\underline{0.20}$ | 0.70 | $\underline{0.14}$ | 0.10 | $\underline{0.02}$ |
|  | 1.00 | $\mathrm{E}\left(\mathrm{R}_{\mathrm{L}}\right)$ | -0.02 | $\mathrm{E}\left(\mathrm{R}_{\mathrm{L}}\right)$ | 0.26 |

- If the risk-free rate is $10 \%$

$$
\begin{aligned}
\text { Risk Premium (U) } & =\mathrm{E}\left(\mathrm{R}_{\mathrm{U}}\right)-\mathrm{R}_{\mathrm{f}} \\
& =26 \%-10 \% \\
& =16 \% \\
\text { Risk Premium (L) } & =\mathrm{E}\left(\mathrm{R}_{\mathrm{L}}\right)-\mathrm{R}_{\mathrm{f}} \\
& =-2 \%-10 \% \\
& =-12 \%
\end{aligned}
$$

## Calculating the Variance

- To calculate the variances of the returns
- Determine the squared deviations from the expected return
- Multiply each possible squared deviation by its probability
- Add up all the products

| State of <br> Economy | P | Return Dev. <br> From ER | Squared Return <br> Dev. From ER | Product |
| :--- | :---: | :---: | :---: | :---: |
| Stock L |  |  |  |  |
| Recession | 0.50 | $-0.20-0.25=-0.45$ | 0.2025 | 0.10125 |
| Boom | $\underline{0.50}$ | $0.70-0.25=0.45$ | 0.2025 | $\underline{0.10125}$ |
|  | 1.00 |  |  | $\sigma_{\mathrm{L}}^{2}=0.2025$ |
| Stock U |  |  | 0.01 | 0.00500 |
| Recession | 0.50 | $0.30-0.20=0.10$ | 0.01 | $\underline{0.00500}$ |
| Boom | $\underline{0.50}$ | $0.10-0.20=-0.10$ |  | $\sigma_{\mathrm{U}}^{2}=0.0100$ |

- Summarizing the expected return and variability information:

| Particulars | Stock L | Stock U |
| :--- | :---: | :---: |
| Expected Return (ER) | $25 \%$ | $20 \%$ |
| Variance,$\sigma^{2}$ | 0.2025 | 0.0100 |
| Standard Deviation, $\sigma$ | $45 \%$ | $10 \%$ |
| Coefficient of variation, $\mathrm{CV}=\sigma / \mathrm{E}(\mathrm{R})$ | 1.8 | 0.5 |

- Stock $L$ has a higher expected return, but $U$ has less risk.
- You could get $70 \%$ return on your investment in L but you could also lose $20 \%$
- An investment in U will always pay at least $10 \%$
- Which of these stocks should you buy?
- For the economy conditions with unequal probabilities, we have summarized the computations in the following table:

| State of <br> Economy | P | Return Dev. <br> From ER | Squared Return <br> Dev. From ER | Product |
| :--- | :---: | :---: | :---: | :---: |
| Stock L |  |  |  |  |
| Recession | 0.80 | $-0.20-(-0.02)=-0.18$ | 0.0324 | 0.02592 |
| Boom | $\underline{0.20}$ | $0.70-(-0.02)=0.72$ | 0.5184 | $\underline{0.10368}$ |
| Stock U | 1.00 |  |  | $\sigma_{\mathrm{L}}=0.12960$ |
| Recession | 0.80 | $0.30-0.26=0.04$ | 0.0016 | 0.00128 |
| Boom | $\underline{0.20}$ | $0.10-0.26=-0.16$ | 0.0256 | $\underline{0.00512}$ |
|  | 1.00 |  |  | $\sigma_{\mathrm{U}}^{2}=0.00640$ |

- Standard deviation for Stock L is

$$
\sigma_{\mathrm{L}}=\sqrt{ } 0.1296=0.36 \text { or } 36 \%
$$

- Standard deviation for Stock U is much smaller

$$
\sigma_{\mathrm{U}}=\sqrt{ } 0.0064=0.08 \text { or } 8 \%
$$

## Portfolios

- Portfolio is the group of assets such as stocks and bonds held by an investor
- Portfolio weights are the percentages of the total portfolio's value that are invested in each portfolio asset.
- If we have $\$ 50$ in one asset and $\$ 150$ in another, then our total portfolio is worth $\$ 200$.
- The percentage of first asset in portfolio is $\$ 50 / 200=0.25$, while the same for the second asset is $\$ 150 / 200=0.75$
- So, the portfolio weights are 0.25 and 0.75
- Let's return to our stocks L and U, where you pay half your money in each, i.e. portfolio weights are 0.50 and 0.50 .

| State of Economy | P | Portfolio Return if State occurs | Product |
| :--- | :---: | ---: | :---: |
| Recession | 0.50 | $0.50 \times-20 \%+0.50 \times 30 \%=5 \%$ | 0.025 |
| Boom | $\underline{0.50}$ | $0.50 \times 70 \%+0.50 \times 10 \%=40 \%$ | $\underline{0.200}$ |
|  | 1.00 |  | ER $(\mathrm{P})=$ |

- Alternatively, Portfolio expected return can be calculated as:

$$
\begin{aligned}
\mathrm{E}\left(\mathrm{R}_{\mathrm{P}}\right) & =0.50 \times \mathrm{E}\left(\mathrm{R}_{\mathrm{L}}\right)+0.50 \times \mathrm{E}\left(\mathrm{R}_{\mathrm{U}}\right) \\
& =0.50 \times 25 \%+0.50 \times 20 \% \\
& =22.5 \%
\end{aligned}
$$

- Suppose we had ' $n$ ' assets in our portfolio, where $n$ is any number, and let ' $x_{i}$ ' stand for percentages of our money in asset $i$, then the expected return is:

$$
\mathrm{E}\left(\mathrm{R}_{\mathrm{P}}\right)=\mathrm{x}_{1} \times \mathrm{E}\left(\mathrm{R}_{1}\right)+\mathrm{x}_{2} \times \mathrm{E}\left(\mathrm{R}_{2}\right)+\ldots \ldots .+\mathrm{x}_{\mathrm{n}} \times \mathrm{E}\left(\mathrm{R}_{\mathrm{n}}\right)
$$

- Variance on a portfolio is not generally a simple combination of the variances of the assets in the portfolio.

| State of <br> Economy | P | Portfolio Return <br> if State occurs | Squared Return <br> Dev. Form ER | Product |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| Recession | 0.50 | $5 \%$ | $(0.05-0.225)^{2}=0.030625$ | 0.0153125 |  |  |
| Boom | $\underline{0.50}$ | $40 \%$ | $(0.40-0.225)^{2}=0.030625$ | $\underline{0.0153125}$ |  |  |
|  | 1.00 |  | $\sigma^{2}{ }_{\mathrm{P}}=$ |  |  | 0.030625 |
|  |  |  | $=\sqrt{0.030625}=17.5 \%$ |  |  |  |

- Applying a different set of weights, i.e. 2/11 (about 18\%) in stock $L$ and other 9/11 (about $82 \%$ ) in stock U :
- If a recession occurs, portfolio returns will be

$$
\mathrm{R}_{\mathrm{P}}=2 / 11 \times-20 \%+9 / 11 \times 30 \%=20.91 \%
$$

- If a boom occurs, portfolio returns will be

$$
R_{P}=2 / 11 \times 70 \%+9 / 11 \times 10 \%=20.91 \%
$$

- The returns are the same, showing zero variance.
- Hence, combining assets into portfolios can substantially alter the risks faced by the investor
- Suppose we have the following projections on three stocks:

| State of <br> Economy |  | Probability of <br> State (P) | Returns |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | Stock A | Stock B | Stock C |  |  |
| Boom | 0.40 | $10 \%$ | $15 \%$ | $20 \%$ |  |
| Bust | 0.60 | $8 \%$ | $4 \%$ | 0 |  |

- What would be the expected return on a portfolio
- with equal amounts invested in each of the assets?
- with half investment in A , remainder divided between B and C ?


## PORTFOLIO

- Suppose we have the following projections on three stocks:

| State of <br> Economy |  | Probability of <br> State (P) | Returns |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | Stock A | Stock B | Stock C |  |  |
| Boom | 0.40 | $10 \%$ | $15 \%$ | $20 \%$ |  |
| Bust | 0.60 | $8 \%$ | $4 \%$ | 0 |  |

- What would be the expected return on a portfolio
- with equal amounts invested in each of the assets?
- with half investment in A , remainder divided between B and C ?
- The expected returns on individual stocks are calculated as
- $\mathrm{E}\left(\mathrm{R}_{\mathrm{A}}\right)=8.8 \%$
- $E\left(R_{B}\right)=8.4 \%$
- $\mathrm{E}\left(\mathrm{R}_{\mathrm{C}}\right)=8.0 \%$
- If a portfolio has equal investment in each asset, the portfolio weights are all the same, $1 / 3$ each in this case. So portfolio expected return is:

$$
\begin{aligned}
\mathrm{E}\left(\mathrm{R}_{\mathrm{P}}\right) & =1 / 3 \times 8.8 \%+1 / 3 \times 8.4 \%+1 / 3 \times 8.0 \% \\
& =8.4 \%
\end{aligned}
$$

- In case of Stock A having half the investment ( $1 / 2$ weight) and remainder divided equally $(1 / 4$ each) between B and C, portfolio returns are:

$$
\begin{aligned}
\mathrm{E}\left(\mathrm{R}_{\mathrm{P}}\right) & =1 / 2 \times 8.8 \%+1 / 4 \times 8.4 \%+1 / 4 \times 8.0 \% \\
& =8.5 \%
\end{aligned}
$$

- Portfolio returns pattern for the case where A has $50 \%$ weights and B and C have $25 \%$ each are:

| State of <br> Economy | Probability of <br> State (P) | Returns |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  | Stock A | Stock B | Stock C | Portfolio |  |
| Boom | 0.40 | $10 \%$ | $15 \%$ | $20 \%$ | $13.75 \%$ |
| Bust | 0.60 | $8 \%$ | $4 \%$ | 0 | $5.00 \%$ |

- Portfolio returns when economy booms is calculated as:

$$
0.50 \times 10 \%+0.25 \times 15 \%+0.25 \times 20 \%=13.75 \%
$$

- Portfolio returns when economy goes bust is calculated as:

$$
0.50 \times 8 \%+0.25 \times 4 \%+0.25 \times 0 \%=5.00 \%
$$

- Variance is thus:

$$
\begin{aligned}
\sigma^{2} & =0.40 \times(0.1375-0.085)^{2}+0.60 \times(0.05-0.085)^{2} \\
& =0.0018375
\end{aligned}
$$

- Standard Deviation is calculated to be $4.3 \%$
- For equally weighted portfolio, standard deviation is about 5.4\%


## Risk

- The true risk of an investment is the unanticipated or surprising part of the return.
- If we always receive exactly what we expect then the investment will be risk-free.
- Systematic Risk
- A risk that influences a large number of assets. It is also called market risk.
- Gross Domestic Product (GDP)
- Interest rate
- Inflation
- Affects wages, cost of supplies, values of the assets owned by company and selling price
- Unsystematic Risk
- A risk that affects a single or at most a small number of assets. Because these risks are unique to individual companies or assets, they are also called unique or asset specific risks.
- The oil strike call in a company will affect that company and perhaps, its primary competitors and suppliers. But it will have little effect on world oil markets and companies not in the oil business
- We know till now that the actual return, R can be broken down into its expected and surprise components:

$$
\begin{aligned}
& \mathrm{R}=\mathrm{E}(\mathrm{R})+\mathrm{U} \\
& \text { Where } \mathrm{U} \text { is the surprise component }
\end{aligned}
$$

- Since the surprise component has a systematic and an unsystematic component, so:

$$
\mathrm{R}=\mathrm{E}(\mathrm{R})+\text { Systematic portion }(\mathrm{m})+\text { Unsystematic portion }(\varepsilon)
$$

- Thus,

$$
\mathrm{R}=\mathrm{E}(\mathrm{R})+\mathrm{m}+\varepsilon
$$

## Diversification and Portfolio Risk

- Suppose if the standard deviation of annual return on a portfolio of 500 large common stocks is, say $20 \%$, does that mean that the standard deviation of annual returns of a certain stock in that portfolio is $20 \%$ ?
- No. Why?


## Diversification and Portfolio Risk

| $\mathbf{( 1 )}$ <br> Number of Stocks <br> in Portfolio | $\mathbf{( 2 )}$ <br> Average Standard Deviation <br> of Portfolio Return | (3) <br> Ratio of Portfolio Standard <br> Deviation to Standard Deviation of <br> Single Stock |
| :---: | :---: | :---: |
| 1 | $49.24 \%$ | 1.00 |
| 10 | 23.93 | 0.49 |
| 50 | 20.20 | 0.41 |
| 100 | 19.69 | 0.40 |
| 300 | 19.34 | 0.39 |
| 500 | 19.27 | 0.39 |
| 1000 | 19.21 | 0.39 |

Source; Meir Statman, "How Many Stocks Make a Diversified Portfolio?" Journal of Financial and Quantitative Analysis 22 (Sep. 1987), pp. 353-64.

- Standard deviation declines as the number of securities is increased.
- By the time we have 100 randomly chosen stocks, the portfolio's standard deviation has declined by about $60 \%$ (from $49 \%$ to $20 \%$ )

- Principle of Diversification
- Benefit in terms of risk reduction from adding securities drops off as we add more and more securities.
- With 10 securities most of the effect is already utilized, and with 30 , there is very little remaining benefit.
- Two key points:
- Some of the riskiness associated with individual assets can be eliminated by forming portfolios.
- The process of spreading an investment across assets (and forming portfolio) is called diversification
- The principle of diversification tells us that spreading an investment across many assets will eliminate some of the risk.
- There is a minimum level of risk that cannot be eliminated simply by diversifying. This minimum level is called non-diversifiable risk.
- If we hold a single stock the value of our investment would fluctuate because of companyspecific events.
- If we hold a large portfolio, some of the stocks in the portfolio will go up and other may go down because of respective company-specific events.
- Net effect on the overall value of the portfolio will be relatively small.


## "Unsystematic risk is essentially eliminated by diversification, so a relatively large portfolio has almost no unsystematic risk."

- Systematic risk cannot be eliminated by diversification, as it affects almost all assets to some degree.
- So the size or type of portfolio will have little effect on systematic risk making it nondiversifiable risk.


## Summarizing

- Total risk $=$ Systematic risk + Unsystematic risk
- Systematic risk is also called non-diversifiable risk or market risk
- Unsystematic risk is also called diversifiable risk, unique risk or asset-specific risk.


## COST OF CAPITAL

- When we say that the required return on an investment is, say, $10 \%$ we mean that the investment will have a positive NPV only if its return exceeds $10 \%$.
- Alternatively, the firm must earn $10 \%$ on the investment just to compensate its investors for the use of the capital needed to finance the project.
- Thus, $10 \%$ is the cost of capital associated with the investment.
- While evaluating a risk-free project, we:
- Look at the capital markets and observe the current rate offered by risk-free investments.
- Use this rate to discount the project's cash flows
- So, the cost of capital here is the risk-free rate.
- If the project is risky, then required return is obviously higher.
- In other words, the cost of capital for a risky project is greater than the risk free rate, and the appropriate discount rate would exceed the risk free rate.
- So, we can use the terms required return, appropriate discount rate and cost of capital interchangeably.
- The cost of the capital associated with an investment depends on the risk of that investment.
- Thus, it is the use of money, not the source of money that matters.
- We know that a firm's overall cost of capital will reflect the required return on the firm's assets as a whole.
- Given that a firm uses both 'debt' and 'equity' capital, this overall cost of capital will be a mixture of the returns needed to compensate its creditors and stockholders.
- Cost of capital will reflect
- Cost of equity capital
- Cost of debt capital


## Cost of Equity

- Recall that under the assumption that the firm's dividend will grow at a constant rate g , the price per share of the stock, P 0 , is:

$$
P_{0}=\frac{D_{0} \times(1+g)}{R_{E}-g}=\frac{D_{1}}{R_{E}-g}
$$

- Where,
- $\mathrm{D}_{0}$ is the dividend just paid
- $\mathrm{D}_{1}$ is the next period's projected dividend
- $R_{E}$ is the required return on the stock
- We can rearrange this to solve for RE as:

$$
R_{E}=\frac{D_{1}}{P_{o}}+\mathrm{g}
$$

- To estimate we need three things; $\mathrm{D}_{0}, \mathrm{P}_{0}$ and g . The first two can be directly observed while g must be estimated.
- Suppose, GSS company paid a dividend of $\$ 4$ per share last year. The stock's current price is $\$ 60$ per share. Assuming that the dividends are estimated to grow steadily at $6 \%$ per year, what is the cost of the capital for GSS?
- Under the dividend growth model:

$$
\begin{aligned}
& \mathrm{D}_{1}=\mathrm{D}_{0} \times(1+\mathrm{g}) \\
& \mathrm{D}_{1}=\$ 4 \times 1.06=\$ 4.24
\end{aligned}
$$

- The cost of equity is:

$$
R_{E}=\frac{D_{1}}{P_{o}}+\mathrm{g}=\frac{4.24}{60}+0.06=13.07 \%
$$

- Estimating g
- To estimate the dividend growth rate, we can use the dividend observations for previous years.
- Alternatively we can use analysts' forecasts of future growth rates.
- Suppose a company has the following dividend pattern in the past years:

| Year | Dividend | Dollar Change | Percentage Change |
| :---: | :---: | :---: | :---: |
| 1999 | $\$ 1.10$ | - | - |
| 2000 | 1.20 | $\$ 0.10$ | $9.09 \%$ |
| 2001 | 1.35 | 0.15 | 12.20 |
| 2002 | 1.40 | 0.05 | 3.70 |
| 2003 | 1.55 | 0.15 | 10.71 |

- The average of the four growth rates is:

$$
(9.09+12.50+3.70+10.71) / 4=9 \%
$$

- This $9 \%$ can be used as an estimate for expected growth rate, g .
- Dividend growth approach
- Easy to understand and use
- Only applicable to companies paying dividends at a constant growth rate
- Estimated cost of equity is very sensitive to estimated growth rate.
- This approach does not necessarily consider risk.


## Cost of Debt

- Cost of debt is the return that firm's creditors demand on the new borrowings.
- This cost of debt can be observed directly or indirectly using the interest rates in the financial markets.
- Alternatively, we can use the firm's bond ratings to estimate the interest rates on newly issued bonds of same rating.
- Coupon rate on the firm's outstanding debt is irrelevant as it relates the firm's cost of debt when the bonds were issued not the cost of debt today.
- Suppose, the GenTech Company issued a 30 -year, $7 \%$ bond 8 years ago. The bond is currently selling for $96 \%$ of its face value, i.e. $\$ 960$. What is GenTech's Cost of debt?
- Using our knowledge of bond valuation, we can calculate that yield to maturity is about $7.37 \%$. So, the GenTech's Cost of debt, RD is $7.37 \%$


## Cost of Preferred Stock

- Preferred stock has a fixed dividend paid every period forever making it a perpetuity.
- Cost of preferred stock, $\mathrm{R}_{\mathrm{P}}$ is:

$$
\mathrm{R}_{\mathrm{P}}=\mathrm{D} / \mathrm{P}_{0}
$$

Where, $\quad \mathrm{D}$ is fixed dividend
$\mathrm{P}_{0}$ is the current price per share of preferred stock

- Alternatively, the cost of preferred stock can be estimated by observing the required return on other similarly rated shares of preferred stock.
- One issue of the preferred stocks of CG Inc. paid $\$ 1.78$ annually and sold for $\$ 25.35$ per share. The other issue paid $\$ 1.72$ annually sold for $\$ 24.90$ per share. What is the cost of preferred stock of CG Inc.?
- Using the first issue, cost of preferred stock was:

$$
\begin{aligned}
& \mathrm{R}_{\mathrm{P}}=\mathrm{D} / \mathrm{P}_{0} \\
& \mathrm{R}_{\mathrm{P}}=\$ 1.78 / 25.35=7.02 \%
\end{aligned}
$$

- Using the second issue, the cost was:

$$
\begin{aligned}
& \mathrm{R}_{\mathrm{P}}=\mathrm{D} / \mathrm{P}_{0} \\
& \mathrm{R}_{\mathrm{P}}=\$ 1.72 / 24.90=6.91 \%
\end{aligned}
$$

- So, the CG Inc.'s cost of preferred stock appears to have been in between 6.9 to $7 \%$.


## Capital Structure Weights

- We can calculate the market value of the firm's equity, E by multiplying the number of shares outstanding by the price per share.
- Market value of firm's debt D can be calculated by multiplying the market price of a single bond by the number of bonds outstanding.
- For multiple bond issues, we repeat this calculation for each and then add up the results.
- For the debt not publicly traded, we observe the yield on similar publicly traded debt and estimate the market value of privately held debt using this yield as discount rate.
- For short term debt we use the book values as estimate of the market value as both should be similar.
- Now the combined market value of debt and equity, V is:

$$
\mathrm{V}=\mathrm{E}+\mathrm{D}
$$

- Dividing both sides by V , we get the percentages of the total capital represented by debt and equity:

$$
100 \%=\mathrm{E} / \mathrm{V}+\mathrm{D} / \mathrm{V}
$$

- These percentages are called capital structure weights
- For example, if the total market value of a company's stocks were calculated as $\$ 200$ million and the total market value of the company's debt were calculated as $\$ 50$ million, then combined value would be $\$ 250$ million.
- Now, $\mathrm{E} / \mathrm{V}=\$ 200 / 250=80 \%$. So $80 \%$ of the firm's financing would be equity and remaining $20 \%$ would be debt.


## The Tax Effect

- Interest paid by corporation is tax deductible but payments to stockholders, such as dividends are not tax deductible.
- This means that the government pays some of the interest.
- So, we need to distinguish between the pretax and after-tax
- Suppose a firm borrows a $\$ 1$ million at $9 \%$. Tax rate is $34 \%$
- The total interest bill will be $\$ 90,000$ per year. This amount is tax deductible, so the $\$ 90,000$ reduces the tax bill by $0.34 \times \$ 90,000=\$ 30,600$
- The after-tax interest bill is thus $\$ 90,000-30,600=\$ 59,400$
- The after-tax interest rate is $\$ 59,400 / 1$ million $=5.94 \%$
- Alternatively, if $\mathrm{T}_{\mathrm{C}}$ stands for corporate tax rate, then the after tax rate that we use for the cost of debt can be written as $R_{D} \times\left(1-T_{C}\right)$.
- So, $9 \% \mathrm{x}(1-0.34)=5.94 \%$


## WEIGHTED AVERAGE COST OF CAPITAL

- To calculate the firm's overall cost of capital, we multiply the capital structures with the associated costs and add up the pieces. The result is called the Weighted Average Cost of Capital (WACC)

$$
\text { WACC }=\frac{E}{V} \times R_{E}+\frac{D}{V} \times R_{D} \times\left(1-T_{C}\right)
$$

- The WACC is the overall return the firm must earn on its existing assets to maintain the value of the stock.
- It is also the required return on any investments by the firm that have the same risks as exiting operations. So, for evaluating the cash flows a proposed expansion project, this is the discount rate to be used.
- For the firm using preferred stocks in its capital structure, the WACC would be:

$$
\text { WACC }=\frac{\mathrm{E}}{\mathrm{~V}} \times \mathrm{R}_{\mathrm{E}}+\frac{\mathrm{P}}{\mathrm{~V}} \times \mathrm{R}_{\mathrm{P}}+\frac{\mathrm{D}}{\mathrm{~V}} \times \mathrm{R}_{\mathrm{D}} \times\left(1-\mathrm{T}_{\mathrm{C}}\right)
$$

- Suppose a company wants to renovate its warehouse distribution system. The plan will cost $\$ 50$ million and is expected to save $\$ 12$ million per year after the taxes over next six years.
- The company has a target debt-equity ratio of $1 / 3$ (i.e. $\mathrm{E} / \mathrm{V}$ is 0.75 and $\mathrm{D} / \mathrm{V}$ is 0.25 ).
- The company has a cost of debt of $10 \%$ and a cost of equity of $20 \%$.
- Assuming a tax rate of $34 \%$, should the company go for the project?
- The Weighted Average Cost of Capital is:

$$
\begin{aligned}
& \text { WACC }=\frac{E}{V} \times R_{E}+\frac{D}{V} \times R_{D} \times\left(1-T_{C}\right) \\
& \text { WACC }=0.75 \times 20 \%+0.25 \times 10 \% \times(1-.34) \\
& \text { WACC }=16.65 \%
\end{aligned}
$$

- Now the NPV will be:

$$
\mathrm{NPV}=-\$ 50+\frac{12}{(1+W A C C)^{1}}+\ldots \ldots \ldots+\frac{12}{(1+W A C C)^{6}}
$$

- Since the cash flows are in the form of an ordinary annuity, we can calculate this NPV using $16.65 \%$ as the discount rate as following:

$$
\begin{aligned}
& \mathrm{NPV}=-\$ 50+\frac{1-\frac{1}{(1+0.1665)^{6}}}{0.1665} \\
& \mathrm{NPV}=-\$ 50+12 \times 3.6222 \\
& \mathrm{NPV}=-\$ 6.53 \text { million }
\end{aligned}
$$

- The negative NPV means that the financial market offers superior projects in the same risk class, so the project should be rejected.


## EMN Corporation

- EMN Corporation has 77.3 million shares of stock outstanding. The book value per share is $\$ 17.62$, but the stock actually sells for $\$ 45.41$.
- The total equity is about $\$ 1.36$ billion on a book value basis but is closer to $\$ 3.51$ billion on a market value basis.
- EMN paid $\$ 1.76$ per share in dividends last year and analysts estimate this dividend to grow by $7 \%$ through next 5 years.
- The estimated cost of equity using dividend growth model is thus:

$$
\begin{aligned}
& R_{E}=\frac{\$ 1.76(1+0.07)}{\$ 45.41}+0.07 \\
& R_{E}=0.115 \text { or } 11.5 \%
\end{aligned}
$$

- EMN has four long term bond issues that account for essentially all of its long term debt. To calculate the cost of debt, we will have to combine these four issues by computing a weighted average.
- The basic information (as in year 2005) is as follows:

| Coupon <br> Rate | Maturity | Book Value <br> (in Millions) | Price <br> (\% of par) | Yield to <br> Maturity |
| :---: | :---: | :---: | :---: | :---: |
| $6.375 \%$ | 2007 | $\$ 496$ | 102.375 | 4.857 |
| 7.25 | 2027 | 496 | 102.007 | 7.067 |
| 7.625 | 2027 | 200 | 113.045 | 6.502 |
| 7.60 | 2030 | 297 | 101.000 | 7.509 |

- To calculate the weighted average cost of debt, we take the percentage of the total debt represented by each issue and multiply by the yield on the issue
- We then add to get the overall weighted average cost of debt. For comparison purpose, we use both book and market values.

| Coupon Rate | Book Value |  | Market Value |  | Yield to Maturity | Book Values | Market Values |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | (in millions) | \% of total | (in millions) | \% of total |  |  |  |
| 6.375\% | \$496 | 0.33 | \$507.8 | 0.33 | 4.857 | 1.62\% | 1.60\% |
| 7.25 | 496 | 0.33 | 506.0 | 0.33 | 7.067 | 2.35 | 2.32 |
| 7.625 | 200 | 0.13 | 226.1 | 0.15 | 6.502 | 0.87 | 0.95 |
| 7.60 | 297 | 0.20 | 300.0 | 0.19 | 7.509 | 1.50 | 1.46 |
| Total | \$1,489 | 1.00 | \$1,539.8 | 1.00 |  | 6.34\% | 6.34\% |

- As these calculations show for EMN, whether market values or book values are used, cost of debt remains $6.34 \%$.
- This is because:
- Market values and book values are similar
- EMN has no preferred stock
- Now to calculate the WACC for EMN on book value basis, we have the following information:
- EMN's equity is worth $\$ 1.362$ billion
- EMN's debt is worth $\$ 1.489$ billion
- Total value is $\$ 2.851$ billion
- So,
- $\mathrm{E} / \mathrm{V}=\$ 1.362 \mathrm{~b} / 2.851 \mathrm{~b}=0.48$, and
- $\mathrm{D} / \mathrm{V}=\$ 1.489 \mathrm{~b} / 2.851 \mathrm{~b}=0.52$
- Assuming a tax rate of $34 \%$, EMN's WACC will is:

$$
\text { WACC }=\frac{E}{V} \times R_{E}+\frac{D}{V} \times R_{D} \times\left(1-T_{C}\right)
$$

$$
\begin{aligned}
& \text { WACC }=0.48 \times 11.15 \%+0.52 \times 6.34 \% \times(1-0.34) \\
& \text { WACC }=7.53 \%
\end{aligned}
$$

- Now to calculate the WACC for EMN on market value basis, we have the following information
- EMN's equity is worth $\$ 3.510$ billion
- EMN's debt is worth $\$ 1.540$ billion
- Total value is $\$ 5.050$ billion
- So,
- $\mathrm{E} / \mathrm{V}=\$ 3.510 \mathrm{~b} / 5.050 \mathrm{~b}=0.70$, and
- $\mathrm{D} / \mathrm{V}=\$ 1.540 \mathrm{~b} / 5.050 \mathrm{~b}=0.30$
- Assuming a tax rate of $34 \%$, EMN's WACC will is:

$$
\begin{aligned}
& \text { WACC }=\frac{\mathrm{E}}{\mathrm{~V}} \times \mathrm{R}_{\mathrm{E}}+\frac{\mathrm{D}}{\mathrm{~V}} \times \mathrm{R}_{\mathrm{D}} \times\left(1-\mathrm{T}_{\mathrm{C}}\right) \\
& \text { WACC }=0.70 \times 11.15 \%+0.30 \times 6.34 \% \times(1-0.34) \\
& \text { WACC }=9.06 \%
\end{aligned}
$$

- Thus, using market value weights, we get $9.06 \%$ for EMN's WACC which is much higher than $7.53 \%$ WACC we get using book value.
- So, using book values can lead to trouble particularly if equity book values are used.
- EMN's market-to-book ratio is about 2.6, so book values significantly overstate the percentage of EMN's financing that comes from debt.


## Summary of Capital Cost

- The Cost of Equity $\mathrm{R}_{\mathrm{E}}$
- Dividend growth model gives:

$$
R_{E}=\frac{D_{1}}{P_{o}}+\mathrm{g}
$$

Where $\mathrm{D}_{1}$ is the expected dividend in one period, g is the dividend growth rate, and $\mathrm{P}_{0}$ is the current stock price.

- The Cost of debt, $\mathrm{R}_{\mathrm{D}}$
- For a firm with publicly held debt, the cost of debt can be measured as the yield to maturity on the outstanding debt. The coupon rate is irrelevant.
- If the firm has no publicly traded debt, then the cost of debt can be measured as the yield to maturity on similarly rated bonds
- The Weighted Average Cost of Capital, WACC
- The firm's WACC is the overall required return on the firm as a whole. It is the appropriate discount rate to use for cash flows similar in risk to the overall firm
- The Weighted Average Cost of Capital, WACC
- WACC is calculated as:

$$
\mathrm{WACC}=\frac{E}{V} \times R_{E}+\frac{D}{V} \times R_{D} \times\left(1-T_{C}\right)
$$

Where, $\quad \mathrm{T}_{\mathrm{C}}$ is the corporate tax rate, E is the market value of the firm's equity,
D is the market value of the firm's debt $\mathrm{V}=\mathrm{E}+\mathrm{D}$.
$\mathrm{E} / \mathrm{V}$ is the percentage of the firm's financing that is equity
$\mathrm{D} / \mathrm{V}$ is the percentage that is debt.

## CAPITAL STRUCTURE

- The guiding principle in choosing the debt-equity ratio is again to choose a course of action that maximizes the value of a share of stock.
- When it comes to capital structure decisions, this is the same thing as maximizing the value of the whole firm.
- Recall, WACC tells us that the firm's overall cost of capital is the weighted average of the costs of various components of the firm's capital structure.
- Usually, while describing WACC, we take the capital structure of the firm as given. But what happens to the cost of capital when we vary the amount of debt financing, or Debt-equity ratio?
- Recall that the WACC is the discount rate appropriate for the firm's overall cash flows.
- Since the values and discount rates move in the opposite directions, we can say that the value of the firm's cash flows (or the value of the firm) is maximized when the WACC is minimized.
- So we can safely say that one capital structure is better than the other if it results in a lower weighted average cost of capital
- Further, a particular debt-equity ratio represents the optimal capital structure if it results in the lowest possible WACC.
- This optimal capital structure is also called firm's target capital structure.


## Financial Leverage

- Financial leverage refers to the extent to which a firm relies on the debt. The more debt financing a firm uses in capital structure, the more financial leverage it employs.
- Financial leverage can dramatically alter the payoffs to the shareholders in the firm, but it may not affect the overall cost of capital.
- While illustrating how financial leverage works, we shall ignore taxes here, and describe the impact of leverage in terms of its effect on earnings per share, EPS and return on equity, ROE.
- For meaningful analysis, we shall use cash flows instead of these accounting figures, but results will be the same
- The TA Corporation currently has no debt in its capital structure. The company is considering a restructuring that would involve issuing debt and using proceeds to buy back some of the outstanding equity.
- The following table presents both current and proposed capital structures.

|  | Current | Proposed |
| :--- | ---: | ---: |
| Assets | $\$ 8,000,000$ | $\$ 8,000,000$ |
| Debt | $\$ 0$ | $\$ 4,000,000$ |
| Equity | $\$ 8,000,000$ | $\$ 4,000,000$ |
| Debt/Equity Ratio | 0 | 1 |
| Share Price | $\$ 20$ | $\$ 20$ |
| Shares Outstanding | 400,000 | 200,000 |
| Interest Rate | $10 \%$ | $10 \%$ |

- EPS and ROE under current capital structure

|  | Recession | Expected | Expansion |
| :--- | ---: | ---: | ---: |
| EBIT | $\$ 500,000$ | $\$ 1,000,000$ | $\$ 1,500,000$ |
| Interest | $\$ 0$ | $\$ 0$ | $\$ 0$ |
| Net Income | $\$ 500,000$ | $\$ 1,000,000$ | $\$ 1,500,000$ |
| EPS | $\$ 1.25$ | $\$ 2.50$ | $\$ 3.75$ |
| ROE | $6.25 \%$ | $12.5 \%$ | $18.75 \%$ |

- EPS and ROE under proposed capital structure

|  | Recession | Expected | Expansion |
| :--- | ---: | ---: | ---: |
| EBIT | $\$ 500,000$ | $\$ 1,000,000$ | $\$ 1,500,000$ |
| Interest | $\$ 400,000$ | $\$ 400,000$ | $\$ 400,000$ |
| Net Income | $\$ 100,000$ | $\$ 600,000$ | $\$ 1,100,000$ |
| EPS | $\$ 0.50$ | $\$ 3.00$ | $\$ 5.50$ |
| ROE | $2.5 \%$ | $15 \%$ | $27.5 \%$ |

- The impact of leverage is evident in the above figures, when we examine the restructuring effect on EPS and ROE.
- Particularly, the variability in both EPS and ROE is much larger under the proposed capital structure, illustrating how financial leverage acts to magnify gains and losses to shareholders.


Disadvantage to debt

- Calculating Break-Even Point
A. With no debt:

$$
\text { EPS }=\mathrm{EBIT} / 400,000
$$

B. With $\$ 4,000,000$ in debt at $10 \%$ :

$$
\text { EPS }=(\text { EBIT }-\$ 400,000) / 200,000
$$

- Calculating Break-Even Point
C. Solve for the break-even EBITBE:

$$
\mathrm{EBIT}_{\mathrm{BE}} / 400,000=\left(\mathrm{EBIT}_{\mathrm{BE}}-\$ 400,000\right) / 200,000
$$

D. With a little algebra:

$$
\begin{aligned}
& \text { EBIT }_{\text {BE }}=\$ 800,000 \\
& \text { And EPS }
\end{aligned}
$$

- The MPD Corporation currently uses no-debt financing and it has decided to go for capital restructuring which would incorporate $\$ 1$ million debt at $9 \%$ debt.
- MPD has 200,000 shares outstanding and the price per share is $\$ 20$.
- If the restructuring is expected to increase EPS, what is the minimum level for EBIT that MPD's management must be expecting?
- To answer we have to calculate the Break-even EBIT
- Under the old capital structure:
EPS = EBIT/200,000
- Under the new capital structure, interest expense will be $\$ 1$ million $\times 0.09=\$ 90,000$
- With $\$ 1$ million proceeds MPD will repurchase $\$ 1$ million $/ 20=50,000$ shares of stock, leaving 150,000 outstanding.
- EPS is thus (EBIT - $\$ 90,000$ )/ 150,000
- Equating both scenarios

$$
\begin{aligned}
& \text { EBIT } / 200,000=(\text { EBIT }-\$ 90,000) / 150,000 \\
& \text { EBIT }=(4 / 3) \times(\text { EBIT }-\$ 90,000) \\
& \text { EBIT }=\$ 360,000
\end{aligned}
$$

- While EPS is $\$ 1.80$ and management of MPD expects this figure to exceed
- From the above discussion, the following conclusions can be drawn :
- The effect of financial leverage depends on the company's EBIT. When EBIT is relatively high, leverage is beneficial.
- Under the unexpected scenario, leverage increases the returns to the shareholders, as measured by ROE and EPS.
- From the above discussion, the following conclusions can be drawn
- Shareholders are exposed to more risk under the proposed capital structure since the EPS and ROE are much more sensitive to changes in EBIT in this case.
- Because of impact that financial leverage has on both the expected return to stockholders and the riskiness of the stock, capital structure is an important consideration


## Homemade Leverage

- The last conclusion is not necessarily correct because the shareholders can adjust the amount of financial leverage by borrowing and lending on their own.
- The use of personal borrowing to alter the degree of financial leverage is called homemade leverage.
- Returning to our discussion of TA Corporation, we will now illustrate that it makes no difference whether or not TA uses the proposed capital structure.
- Proposed Capital Structure

|  | Recession | Expected | Expansion |
| :--- | ---: | ---: | ---: |
| EPS | $\$ 0.50$ | $\$ 3.00$ | $\$ 5.50$ |
| Earning for 100 shares | $\$ 50.00$ | $\$ 300.00$ | $\$ 550.00$ |
| Net Cost $=100$ share at $\$ 20=\$ 2,000$ |  |  |  |

- Original capital structure and homemade leverage

|  | Recession | Expected | Expansion |
| :--- | ---: | ---: | ---: |
| EPS | $\$ 1.25$ | $\$ 2.50$ | $\$ 3.50$ |
| Earning for 200 shares | $\$ 250.00$ | $\$ 500.00$ | $\$ 750.00$ |
| Less Interest on $\$ 2,000$ at $10 \%$ | $\$ 200.00$ | $\$ 200.00$ | $\$ 200.00$ |
| Net Earnings | $\$ 50.00$ | $\$ 300.00$ | $\$ 550.00$ |
| Net Cost $=200$ shares at $\$ 20-$ amount borrowed |  |  |  |
| $\quad=\$ 4,000-\$ 2,000=\$ 2,000$ |  |  |  |

- The proposed capital structure results in a debt-equity ratio of 1 .
- To replicate this capital structure at the personal level, the stockholder must borrow enough to create this same debt-equity ratio of 1 .


## Unlevering

- Now, under the condition that TA management adopted the proposed capital structure, suppose, an investor who owned 100 shares preferred the original capital structure
- To create leverage, investors borrow on their own, while to unlever investors must loan out the money.
- In TA, the corporation borrowed an amount equal to half its value. The investor can unlever the stock by simply solving loaning out the money in the same proportion
- The investor sells 50 shares for $\$ 1,000$ total and then loans out $\$ 1,000$ at $10 \%$.

|  | Recession | Expected | Expansion |
| :--- | ---: | ---: | ---: |
| EPS (Proposed Structure) | $\$ 0.50$ | $\$ 3.00$ | $\$ 5.50$ |
| Earning for 50 shares | $\$ 25.00$ | $\$ 150.00$ | $\$ 275.00$ |
| Plus Interest on $\$ 1,000$ at 10\% | $\$ 100.00$ | $\$ 100.00$ | $\$ 100.00$ |
| Total Payoff | $\$ 125.00$ | $\$ 250.00$ | $\$ 375.00$ |

## M\&M PROPOSITIONS

- We have seen that corporate borrowing is relatively less significant when it comes to corporate structure, because investors can borrow or lend on their own.
- So, in our TA Corporation, the stock price remains almost the same whichever capital structure company chooses.
- This result is based upon a famous argument by two noble laureates, Franco Modigliani and Merton Miller (commonly known as M\&M).
- We shall discuss the two propositions presented by M\&M.
- The 1 st proposition states that it is completely irrelevant how a firm chooses to arrange its finances.
- Imagine two firms having identical assets and operations depicted on the left hand side of the balance sheet.
- But the right hand side is different because the two firms finance their operations differently.
- In this case we can view the capital structure question in terms of a pie model.

- The size of the pie does not depend on how it is sliced.
- Although changing the capital structure may not change the firm's total value, it does cause important changes in the firm's debt and equity.
- Now we examine the firm which changes its debt-equity ratio; and for simplification we ignore taxes here.
- We know that:

$$
\begin{aligned}
& \text { WACC }=\frac{E}{V} \times R_{E}+\frac{D}{V} \times R_{D} \\
& \text { Where } V=E+D
\end{aligned}
$$

- We also know that WACC can be interpreted as the required return on the firm's overall assets $\left(\mathrm{R}_{\mathrm{A}}\right)$. Thus

$$
R_{A}=\frac{E}{V} \times R_{E}+\frac{D}{V} \times R_{D}
$$

- Rearranging

$$
\mathrm{R}_{\mathrm{E}}=\mathrm{R}_{\mathrm{A}}+\left(\mathrm{R}_{\mathrm{A}}-\mathrm{R}_{\mathrm{D}}\right) \mathrm{x}(\mathrm{D} / \mathrm{E})
$$

- This 2nd proposition tells us that the cost of equity depends on three things
- The required return on firm's assets $R_{A}$
- The firm's cost of debt $R_{D}$, and
- The firm's debt-equity ratio $\mathrm{D} / \mathrm{E}$


## Cost of Capital

(\%)


Debt-equity
ratio (D/E)

- As the firm raises its debt-equity ratio, the increase in leverage raises the risk of the equity and therefore the required return or cost of equity (RE)
- WACC remains the same, supporting M\&M proposition 1
- RCD Corporation has a WACC of $12 \%$ (ignoring taxes). It can borrow at $8 \%$.
- Assuming that RCD has a target capital structure of $80 \%$ equity and $20 \%$ debt, what is its cost of equity?
- What is the cost of equity if the target capital structure is $50 \%$ equity?
- Calculate WACC in both cases to verify it remains the same.
- According to M\&M proposition 2, the cost of equity, RE, is:

$$
R_{E}=R_{A}+\left(R_{A}-R_{D}\right) \times(D / E)
$$

- In the 1 st case, debt-equity ratio is $.2 / .8=.25$, so the cost of equity is

$$
\begin{aligned}
& \mathrm{R}_{\mathrm{E}}=12 \%+(12 \%-8 \%) \times 0.25 \\
& \mathrm{R}_{\mathrm{E}}=13 \%
\end{aligned}
$$

- In the second case, debt-equity comes out to be 1.0 , so the cost of equity is $16 \%$
- Now assuming equity financing is $80 \%$, the cost of equity is $13 \%$ and tax rate is zero, WACC is

$$
\begin{aligned}
& \text { WACC }=\frac{E}{V} \times R_{E}+\frac{D}{V} \times R_{D} \\
& \text { WACC }=0.80 \times 13 \%+0.20 \times 8 \% \\
& \text { WACC }=12 \%
\end{aligned}
$$

- In the second case, equity financing is $50 \%$, the cost of equity is $16 \%$. WACC is

$$
\begin{aligned}
& \text { WACC }=\frac{E}{V} \times R_{E}+\frac{D}{V} \times R_{D} \\
& \text { WACC }=0.50 \times 16 \%+0.50 \times 8 \% \\
& \text { WACC }=12 \%
\end{aligned}
$$

- So, WACC is $12 \%$ in both cases.


## Business and Financial Risk

- M\&M proposition 2 shows that the firm's cost of equity can be broken into two components
- The required return on firm's assets, $\mathrm{R}_{\mathrm{A}}$ depends on the nature of the firm's operating activities
- The risk inherent in the firm's operations is called the business risk, and we know that this business risk depends on the systematic risk of the firm's assets.
- M\&M proposition 2 shows that the firm's cost of equity can be broken into two components - $\left(R_{A}-R_{D}\right) \times(D / E)$ is determined by the firm's financial structure.
- For an all equity firm, this component is zero.
- The increase in debt financing raises the required return on equity because the risk born by the investors increases.
- This extra risk is called financial risk.


## Corporate Taxes \& Capital Structure

- Debt features
- Interest paid on debt is tax deductible; a benefit for the firm
- Failure to meet debt financing may lead to bankruptcy; a cost of debt financing
- To examine the effect of corporate taxes, we consider two firms (Firm U \& Firm L), identical on left side of balance sheet. So their assets and operations are the same.
- Assuming EBIT to be $\$ 1,000$ every year for both firms, the difference is that Firm $L$ has issued $\$ 1,000$ worth of perpetual bonds at $8 \%$ interest every year.

|  | Firm U | Firm L |
| :--- | ---: | ---: |
| EBIT | $\$ 1,000$ | $\$ 1,000$ |
| Interest | 0 | 80 |
| Taxable Income | $\$ 1,000$ | $\$ 920$ |
| Taxes $(30 \%)$ | 300 | 276 |
| Net Income | $\$ 700$ | $\$ 644$ |

- To simplify, we assume
- Depreciation is zero
- Capital spending is zero
- There are no additions to Net Working Capital
- Thus cash flow from assets is simply equal to EBIT - Taxes.

| Cash Flow from Assets | Firm U | Firm L |
| :--- | ---: | ---: |
| EBIT | $\$ 1,000$ | $\$ 1,000$ |
| - Interest | 300 | 276 |
| Total | $\$ 700$ | $\$ 724$ |

- Now the cash flow to stockholders and bondholders are:

| Cash Flow | Firm U | Firm L |
| :--- | ---: | ---: |
| To Stockholders | $\$ 700$ | $\$ 644$ |
| To Bondholders | 0 | 80 |
| Total | $\$ 700$ | $\$ 724$ |

- Total cash flow to Firm L is $\$ 24$ more, as its tax bill is $\$ 24$ less.
- Interest being tax deductible has generated a tax saving equal to interest payment multiplied by tax rate, i.e.

$$
\$ 80 \times 0.30=\$ 24
$$

- This tax saving is called the Interest Tax Shield
- Since debt is perpetual, the same $\$ 24$ shield will be generated every year forever.
- Since Firm L's cash flow is always $\$ 24$ greater, its worth is more than Firm U by the value of this $\$ 24$ perpetuity.
- Because the tax shied is generated by paying interest, it has the same risk as the debt, and $8 \%$ is therefore the appropriate discount rate.
- The value of the tax shield is:

$$
\begin{aligned}
\mathrm{PV} & =\frac{\$ 24}{0.08}=\frac{0.30 \times \$ 1,000 \times 0.08}{0.08} \\
& =0.30 \times \$ 1,000=\$ 300
\end{aligned}
$$

## Corporate Taxes \& Capital Structure

- So:

$$
\begin{aligned}
\text { PV of interest tax shield } & =\left(\mathrm{T}_{\mathrm{C}} \times \mathrm{D} \times \mathrm{R}_{\mathrm{D}}\right) / \mathrm{R}_{\mathrm{D}} \\
& =\mathrm{T}_{\mathrm{C}} \times \mathrm{D}
\end{aligned}
$$

- We may conclude here that the value of Firm $\mathrm{L}, \mathrm{V}_{\mathrm{L}}$, exceeds the value of the firm $\mathrm{U}, \mathrm{V}_{\mathrm{U}}$, by the present value of the interest tax shield, $\mathrm{T}_{\mathrm{C}} \times \mathrm{D}$.
- M\&M Proposition 1 With taxes therefore states that:

$$
\mathrm{V}_{\mathrm{L}}=\mathrm{V}_{\mathrm{U}}+\mathrm{T}_{\mathrm{C}} \times \mathrm{D}
$$



## M\&M Summary

- The no-tax case
- Proposition 1: the value of the firm levered $\left(\mathrm{V}_{\mathrm{L}}\right)$ is equal to the firm unlevered $\left(\mathrm{V}_{\mathrm{U}}\right)$ :

$$
\mathrm{V}_{\mathrm{L}}=\mathrm{V}_{\mathrm{U}}
$$

- Implications of proposition 1:
- A Firm's capital structure is irrelevant
- A firms WACC is the same no matter what mixture of debt and equity is used to finance the firm
- The no-tax case
- Proposition 2: The cost of equity, $\mathrm{R}_{\mathrm{E}}$ is:

$$
\mathrm{R}_{\mathrm{E}}=\mathrm{R}_{\mathrm{A}}+\left(\mathrm{R}_{\mathrm{A}}-\mathrm{R}_{\mathrm{D}}\right) \times \mathrm{D} / \mathrm{E}
$$

Where $R_{A}$ is the WACC, $R_{D}$ is the cost of debt and $D / E$ is debt-equity ratio

- Implications of Proposition 2:
- Cost of equity rises as the firm increases its use of debt financing
- The risk of the equity depends on:
- riskiness of firms operations (business risk) which determines $R_{A}$
- Degree of financial leverage (financial risk) determined by $\mathrm{D} / \mathrm{E}$
- The Tax Case
- Proposition 1 with taxes: the value of the firm levered $\left(\mathrm{V}_{\mathrm{L}}\right)$ is equal to the value of the firm unlevered $\left(\mathrm{V}_{\mathrm{U}}\right)$ plus the present value of the interest tax shield.

$$
\mathrm{V}_{\mathrm{L}}=\mathrm{V}_{\mathrm{U}}+\mathrm{T}_{\mathrm{C}} \times \mathrm{D}
$$

- The Tax Case
- Implications of Proposition 1 with taxes
- Debt financing is highly advantageous and in the extreme a firm's optimal capital structure is $100 \%$ debt.
- A firm's WACC decreases as the firm relies more heavily on debt financing


## BANKRUPTCY COSTS

- Direct Bankruptcy Costs
- The costs that are directly associated with bankruptcy, such as legal and administrative expenses
- Indirect Bankruptcy Costs
- The costs of avoiding a bankruptcy filing incurred by a financially distress firm
- Financial Distress Costs
- The direct and indirect costs associated with going bankrupt or experiencing financial distress


## Static Theory of Capital Structure

- A firm borrows up to the point where tax benefit from an extra dollar in debt is exactly equal to the cost that comes from the increased probability of financial distress.



## Optimal Capital Structure

- The maximum value of the firm, $\mathrm{V}_{\mathrm{L}} *$ is reached at a debt level of $\mathrm{D}^{*}$, so this is the optimal amount of borrowing
- Alternatively, the firm's capital structure is composed of $\mathrm{D}^{*} / \mathrm{V}_{\mathrm{L}}{ }^{*}$ in debt and (1- $\left.\mathrm{D}^{*} / \mathrm{V}_{\mathrm{L}}{ }^{*}\right)$ in equity.
- The difference between the value of firm in static theory and the M\&M value of the firm with taxes is the loss in value from the possibility of financial distress
- The difference between the static theory value of the firm and the M\&M value with no taxes is the gain from leverage, net of distress costs.
- We know that the capital structure that maximizes the value of the firm is also the one that minimizes the cost of capital.
- We illustrate this discussion focusing on the three cases:


## Case 1

With no taxes and bankruptcy costs, the value of the firm and its weighted average cost of capital (WACC) are not affected by capital structure.



## Case 2

With corporate taxes and no bankruptcy costs the value of the firm increases and the WACC decreases as the amount of debt goes up.



## Case 3

With corporate taxes and bankruptcy costs, the value of the firm, $\mathrm{V}_{\mathrm{L}}$, reaches a maximum at $\mathrm{D}^{*}$, the optimal amount of borrowing. At the same time, the WACC is minimized at $\mathrm{D}^{*} / \mathrm{E}^{*}$


## Some Managerial Recommendations

- Tax benefit from leveraging is only important to the firms that are in a tax-paying position. Firms with substantial losses will get little value from the interest tax shield.
- Firms that have substantial tax shields from other sources, such as depreciation, will get less benefit from leverage.
- Firms with a greater risk of experiencing financial distress will borrow less than the firms with a lower risk.
- The cost of financial distress depends primarily on the firm's assets and it will be determined by how easily the ownership of those assets can be transferred
- Tangible assets vs. Intangible assets


## Net Working Capital

- Net working capital (NWC) is the difference between the current assets and the current liabilities. Often the short-term financing is called net working capital management.
- The difference between short- and long- term financing is the timing of the cash flows.
- The questions to address under short-term financing are
- What is the reasonable level of cash to keep on hand for paying bills?
- How much the firm should borrow in short term?
- How much credit should be extended to the customers?
- Current assets
- Cash and other assets that are expected to convert to cash within one year
- Presented on balance sheet in order of their liquidity
- Cash \& equivalents
- Marketable securities
- Accounts receivables
- Inventories
- Current liabilities
- Obligations that are expected to require cash payment within one year
- Accounts payable
- Expenses payable (including accrued wages and taxes)
- Notes payable

The basic balance sheet identity can be written as:

$$
\text { NWC }+ \text { Fixed assets }=\text { Long term debt }+ \text { Equity }
$$

While,
NWC $=$ (Cash + other current assets)

- Current liabilities

$$
\begin{aligned}
& \text { So, } \\
& \text { Cash }=\quad \text { Long-term debt } \\
&+ \text { Equity } \\
&+ \text { Current liabilities } \\
&- \text { Current assets other than cash } \\
& \text { Fixed assets }
\end{aligned}
$$

- Activities that Increase Cash (sources of cash)
- Increasing long-term debt
- Increasing equity
- Increasing current liabilities
- Decreasing current assets other than cash
- Decreasing fixed assets
- Activities that decrease cash (uses of cash)
- Decreasing long term debt
- Decreasing equity
- Decreasing current liabilities
- Increasing current assets other than cash
- Increasing fixed assets
- Sources of cash always involve increasing a liability (or equity) account or decreasing an asset account
- Uses of cash involve decreasing a liability or increasing assets.
- Is it a source or use?
- If accounts payable go up by $\$ 100$
- If accounts receivable up by $\$ 100$


## OPERATING CYCLE AND CASH CYCLE

For a typical manufacturing firm, short run activities might consist of the following sequence of events and decisions:

| Events | Decisions |
| :--- | :--- |
| 1. Buying raw materials | 1. How much inventory to order? |
| 2. Paying cash | 2. Whether to borrow or draw down cash balances? |
| 3. Manufacturing the product | 3. What choice of production technology to use? |
| 4. Selling the product | 4. Whether credit should be extended to a particular customer? |
| 5. Collecting cash | 5. How to collect? |

- These activities are
- Unsynchronized because payment for purchases may not happen at the same time as the receipts of sales.
- Uncertain because future sales and costs cannot be precisely predicted. Consider the following chronological events.
- Consider the following chronological events:

| Day | Activity | Cash Effect |
| :---: | :--- | :---: |
| 0 | Acquire inventory on credit | None |
| 30 | Pay for inventory | $-\$ 1,000$ |
| 60 | Sell inventory on credit | None |
| 105 | Collect on sale | $+\$ 1,400$ |

- Operating cycle
- The time period between the acquisition of inventory and the collection of cash from receivables. (in our example, it is 105 days)
- Inventory period
- The time it takes to acquire and sell inventory. (in our example, it is 60 days)
- Accounts receivable period
- The time between sale of inventory and collection of receivable. (here, it is 45 days)
- So, we can describe the operating cycle as:

$$
\begin{aligned}
& \text { Operating Cycle }
\end{aligned}=\text { Inventory period }+ \text { Accounts receivable period } ~ 子 105 \text { days } \quad=60 \text { days }+45 \text { days }
$$

- An operating cycle describes how a product moves through the current asset accounts
- It begins life as inventory
- Converted to a receivable when it is sold
- Converted to cash when we collect from the sale
- Accounts payable period
- The time between receipt of inventory and payment for it. (In our example we pay for inventory purchased after 30 days.)
- Cash cycle
- The time between cash disbursement and cash collection. (We spend cash on day 30, but don't collect until day 105. so we have to arrange finances $\$ 1,000$ for $105-30=75$ days)
- So, we can describe the cash cycle as:

$$
\begin{aligned}
& \text { Cash Cycle }=\text { Operating cycle }- \text { Accounts payable period } \\
& 75 \text { days }=105 \text { days }-30 \text { days }
\end{aligned}
$$



- The gap between the short-term inflows and outflows can be filled either by borrowing or by holding a liquidity reserve in the form of cash or marketable securities.
- Alternatively, the gap can be shortened by changing the inventory, receivable and payable periods.


## Managing Operating Cycle

| Title of Manager | Duties Relating Short-Term Financial <br> Management | Assets/Liabilities <br> influenced |
| :--- | :--- | :--- |
| Cash Manager | Collection, concentration, disbursement; <br> short-term investment; short-term <br> borrowing; bank relations | Cash, marketable securities, <br> short-term loans |
| Credit Manager | Monitoring and control of accounts <br> receivable; credit policy decisions | Accounts receivable |
| Marketing <br> Manager | Credit policy decisions | Accounts receivable |
| Purchase Manager | Decisions on purchases, supplies; may <br> negotiate payment terms | Inventory, accounts payable |
| Production <br> Manager | Setting production schedules and materials <br> Requirements | Inventory, accounts payable |
| Payables Manager | Decisions on payment policies and on <br> whether to take discounts | Accounts payable |
| Controller | Accounting info. on cash flows; <br> reconciliation of A/C receivable; application <br> of payments to A/C receivable | Accounts receivable, |
| accounts payable |  |  |

- Following is some of the balance sheet information (in thousands):

| Item | Beginning | Ending | Average |
| :--- | ---: | ---: | ---: |
| Inventory | $\$ 2,000$ | $\$ 3,000$ | $\$ 2,500$ |
| Accounts receivable | 1,600 | 2,000 | 1,800 |
| Accounts payable | 750 | 1,000 | 875 |

- Also the most recent income statement gives the following (in thousands):

$$
\left.\begin{array}{lc}
\text { Net Sales } & \$ 11,500 \\
\text { Cost of Goods Sold } & 8,200
\end{array}\right] \begin{aligned}
& \text { Cost of Goods Sold } \\
& \text { Average Inventory }
\end{aligned}
$$

$$
\begin{aligned}
& \text { Inventory Turnover }=\frac{8,200,000}{2,500,000} \\
& \text { Inventory Turnover }=3.28 \text { times } \\
& \text { Inventory Period }=\frac{365 \text { Days }}{\text { Inventory Turnover }} \\
& \text { Inventory Period }=\frac{365}{3.28} \\
& \text { Inventory Period }=111.3 \text { days }
\end{aligned}
$$

- It means inventory sat for 111 days before it was sold.
- Now, assuming that all sales were credit sales, the receivables turnover is:

$$
\begin{aligned}
& \text { Receivable Turnover }=\frac{\text { Credit Sales }}{\text { Average A/C Receivable }} \\
& \text { Receivable Turnover }=\frac{11,500,000}{1,800,000} \\
& \text { Receivable Turnover }=6.4 \text { times } \\
& \text { Receivable Period }=\frac{365 \text { Days }}{\text { Receivable Turnover }} \\
& \text { Receivable Period }=\frac{365}{6.4} \\
& \text { Receivable Period }=57 \text { days }
\end{aligned}
$$

- The receivable period is also called days' sales in receivables or average collection period.
- It tells that our customers took an average 57 days to pay.
- The operating cycle is the sum of the inventory and receivable periods

Operating Cycle $=$ Inventory period + Receivable period Operating Cycle $=111+57=168$ days

- So on average, 168 days elapse between the time we acquire inventory and having sold it, collect for sale.

$$
\begin{aligned}
& \text { Payables Turnover }=\frac{\text { Cost of Goods Sold }}{\text { Average Payables }} \\
& \text { Payables Turnover }=\frac{8,200,000}{875,000} \\
& \text { Payables Turnover }=9.4 \text { times } \\
& \text { Payables Period }=\frac{365 \text { Days }}{\text { Payables Turnover }} \\
& \text { Payables Period }=\frac{365}{9.4} \\
& \text { Payables Period }=39 \text { days }
\end{aligned}
$$

- Cash cycle is the difference between the operating cycle and the payables period:

Cash Cycle $=$ Operating Cycle $-\mathrm{A} / \mathrm{C}$ Payables Period

$$
\text { Cash Cycle }=168-39=129 \text { days }
$$

- So on average, there is a 129 -day delay from the time we pay for merchandise to the time we collect on the sales.
- SP Company has the following accounting figures:

| Item | Beginning | Ending |
| :--- | ---: | ---: |
| Inventory | $\$ 5,000$ | $\$ 7,000$ |
| Accounts receivable | 1,600 | 2,400 |
| Accounts payable | 2,700 | 4,800 |

- Credit sales for the year were $\$ 50,000$ and cost of goods sold was $\$ 30,000$
- Calculating the three turnovers:

> Inventory turnover $=\$ 30,000 / 6,000=5$ times
> Receivable turnover $=\$ 50,000 / 2,000=25$ times
> Payable turnover $=\$ 30,000 / 3,750=8$ times

- Now using these to get the various periods:

$$
\begin{aligned}
& \text { Inventory period }=365 / 5=73 \text { days } \\
& \text { Receivable period }=365 / 25=14.6 \text { days } \\
& \text { Payable period }=365 / 8=45.6 \text { days }
\end{aligned}
$$

- Operating Cycle and Cash Cycle:

Operating cycle $=73+14.6=87.6$ days
Cash cycle $=87.6-45.6=42$ days

## Interpreting Cash Cycle

- These calculations reveal that cash cycle
- depend on the inventory, receivable and payables turnover
- Increases if inventory and receivables periods get longer
- Decreases if payable period is lengthened
- Most firms have a positive cash cycle and they require more financing for inventories and receivables for longer cash cycle.
- The link between the firm's cash cycle and its profitability is provided by Total Assets Turnover (Sales/Total assets).
- The higher this ratio is; the greater are the firm's accounting return on assets (ROA) and return on equity (ROE).
- So, the shorter the cash cycle is, the lower is the firm's investment in inventories and receivables.
- Thus, the firm's total assets and lower and total turnover is higher.


## SHORT-TERM FINANCIAL POLICY

- Size of investments in current assets
- Flexible policy
- maintain a high ratio of current assets to sales
- Restrictive policy
- maintain a low ratio of current assets to sales
- Financing of current assets
- Flexible policy
- less short-term debt and more long-term debt
- Restrictive policy
- more short-term debt and less long-term debt
- If policies are flexible with regard to current assets
- Keep larger cash/marketable securities balances
- Larger investments in inventory
- More liberal credit terms, thus higher accounts receivable
- If policies are restrictive with regard to current assets
- Keep lower cash/marketable securities balances
- Make smaller investments in inventory
- Tight or no credit sales, minimal accts. receivable
- The optimum level of investment in short term assets depends on a trade-off between the costs of a restrictive policy against the costs of a flexible one.
- Current assets holdings are highest with a flexible short term financial policy and lowest with a restrictive one.
- So flexible policies are costly as they require greater investment in current assets.
- However expected future cash-flows are higher too.
- A restrictive short-term financial policy reduces future sales level than under flexible policy.
- Managing short-term assets involves a trade-off between carrying costs and shortage costs


## - Carrying costs

- Increase with increased levels of current assets
- are the costs to store and finance the assets,
- are the opportunity costs associated with current assets (inventories vs. short term investments)


## - Shortage costs

- decrease with increased levels of current assets
- are the costs to replenish assets
- Trading or order costs (avoiding stock-outs or cash-outs through more frequent orders, etc.)
- Costs related to lack of safety reserves, i.e., lost sales and customers and production stoppages.


CA* represents the optimal amount of current assets. Holding this amount minimizes total costs.

## A. Flexible Policy



A flexible policy is most appropriate when carrying costs are low relative to shortage costs.
B. Restrictive policy


A restrictive policy is most appropriate when carrying costs are high relative to shortage costs.

## Alternative Financing Policies

- Focusing on the financing side of the picture, the total asset requirements for a growing firm may exhibit change over time for many reasons
- A general growth trend
- Seasonal variation around the trend
- Unpredictable day-to-day and month-to-month fluctuations
- Temporary current assets
- Sales or required inventory build-ups are often seasonal.
- The additional current assets carried during the "peak" time.
- The level of current assets will decrease as sales occur.
- Permanent current assets
- Firms generally need to carry a minimum level of current assets at all times.
- These assets are considered "permanent" because the level is constant, not because the assets aren't sold.



## Policy F



Policy F always implies a short-term cash surplus and a large investment in cash and marketable securities.

## Policy R



Policy R uses long-term financing for permanent asset requirements only and short-term borrowing for seasonal variations.

## Which is the Best Policy?

- Cash Reserves
- Pros - firms will be less likely to experience financial distress and are better able to handle emergencies or take advantage of unexpected opportunities
- Cons - cash and marketable securities earn a lower return and are zero NPV investments
- Maturity Hedging
- Try to match financing maturities with asset maturities
- Finance temporary current assets with short-term debt

Finance permanent current assets and fixed assets with long-term debt and equity

- Interest Rates
- Short-term rates are normally lower than long-term rates, so it may be cheaper to finance with short-term debt.
- Firms can get into trouble if rates increase quickly or if it begins to have difficulty making payments - may not be able to refinance the short-term loans.


With a compromise policy, the firm keeps a reserve of liquidity which it uses to initially finance seasonal variations in current asset needs. Short-term borrowing is used when the reserve is exhausted.

## Cash Budget

- Forecast of cash inflows and outflows over the next short-term planning period
- Primary tool in short-term financial planning
- Helps determine when the firm should experience cash surpluses and when it will need to borrow to cover working-capital costs
- Allows a company to plan ahead and begin the search for financing before the money is actually needed
- Records the estimates of cash receipts (cash in) and disbursements (cash out).
- PT Inc. specializes in toys and receives all income from sales.
- Sales estimates (in millions)
- $\mathrm{Q} 1=500 ; \mathrm{Q} 2=600 ; \mathrm{Q} 3=650 ; \mathrm{Q} 4=800 ; \mathrm{Q} 1$ next year $=550$
- Accounts receivable
- Beginning receivables $=\$ 250$
- Average collection period $=30$ days
- Accounts payable
- Purchases $=50 \%$ of next quarter's sales
- Beginning payables $=125$
- Accounts payable period is 45 days
- Other expenses
- Wages, taxes and other expense are $25 \%$ of sales
- Interest and dividend payments are $\$ 50$
- A major capital expenditure of $\$ 200$ is expected in the second quarter
- The initial cash balance is $\$ 100$ and the company maintains a minimum balance of $\$ 50$
- Average Collection Period $=30$ days, this implies that $2 / 3$ of sales are collected in the quarter made and the remaining $1 / 3$ are collected the following quarter
- Beginning receivables of $\$ 250$ will be collected in the first quarter


## Cash Collections

|  | Q1 | Q2 | Q3 | Q4 |
| :--- | ---: | ---: | ---: | ---: |
| Beginning Receivables | $\$ 250$ | $\$ 167$ | $\$ 200$ | $\$ 217$ |
| Sales | 500 | 600 | 650 | 800 |
| Cash Collections | 583 | 567 | 633 | 750 |
| Ending Receivables | 167 | 200 | 217 | 267 |

- Payables period is 45 days, so half of the purchases will be paid for each quarter and the remaining will be paid the following quarter
- Beginning payables $=\$ 125$
- An additional payment of $\$ 125$ will be made to a vendor in second quarter, which is related to some previous transaction.


## Cash Disbursement

|  | Q1 | Q2 | Q3 | Q4 |
| :--- | ---: | ---: | ---: | ---: |
| Payment of accounts | $\$ 275$ | $\$ 438$ | $\$ 362$ | $\$ 338$ |
| Wages, taxes and other expenses | 125 | 150 | 163 | 200 |
| Capital expenditures | - | 200 | - | - |
| (Interest and dividend payments) | 50 | 50 | 50 | 50 |
| Total cash disbursements | $\$ 450$ | $\$ 838$ | $\$ 575$ | $\$ 588$ |

## Net Cash Inflow

|  | Q1 | Q2 | Q3 | Q4 |
| :--- | ---: | ---: | ---: | ---: |
| Total cash collections | $\$ 583$ | $\$ 567$ | $\$ 633$ | $\$ 750$ |
| Total cash disbursements | 450 | 838 | 575 | 588 |
| Net cash inflow | $\$ 133$ | $-\$ 271$ | $\$ 58$ | $\$ 162$ |

## Cash Balances

|  | Q1 | Q2 | Q3 | Q4 |
| :--- | ---: | ---: | ---: | ---: |
| Beginning Cash Balance | $\$ 100$ | $\$ 233$ | $-\$ 38$ | $\$ 20$ |
| Net cash inflow | 133 | -271 | 58 | 162 |
| Ending cash balance | 233 | -38 | 20 | 182 |
| Minimum cash balance | -50 | -50 | -50 | -50 |
| Cumulative surplus (deficit) | $\$ 183$ | $-\$ 88$ | $-\$ 30$ | $\$ 132$ |

## SHORT-TERM BORROWING

- Unsecured Loans
- Line of credit
- Committed vs. Non committed
- Committed being formal arrangements involving a commitment fee
- Non committed being an informal channel involving lesser paper work
- Revolving credit arrangement
- Open for two or more years
- Letter of credit
- Secured Loans
- Accounts receivable financing
- Assigning
- Borrower is responsible even if receivables are not collected
- Factoring
- Receivable is discounted and sold to lender (factor) who then bears the risk of default
- Maturity Factoring
- Factor forwards the money on an agreed-upon future date
- Secured Loans
- Inventory loans
- Blanket inventory lien
- Gives a lien against all the inventories
- Trust receipt
- Borrower holds specific inventory in "trust" for the lender; also called floor planning
- Field warehouse financing
- An independent company specialized in inventory management acts as control agent
- Other Sources
- Commercial Paper
- Short term notes issued by large and highly rated firms
- Trade Credit
- Increase the accounts payable period, delaying the payments


## A Short-Term Financial Plan

- PT Inc. arranges to borrow any needed funds on short term basis.
- The interest rate is $20 \%$ APR, calculated on quarterly basis $(20 \% / 4=5 \%)$
- There is no starting short term debt.

|  | Q1 | Q2 | Q3 | Q4 |
| :--- | ---: | ---: | ---: | ---: |
| Beginning Cash Balance | $\$ 20$ | $\$ 60$ | $-\$ 50$ | $\$ 5$ |
| Net cash inflow | 40 | -110 | 55 | -15 |
| Ending cash balance | $\$ 60$ | $-\$ 50$ | $\$ 5$ | $-\$ 10$ |
| Minimum cash balance | -10 | -10 | -10 | -10 |
| Cumulative surplus(deficit) | $\$ 50$ | $-\$ 60$ | $-\$ 5$ | $-\$ 20$ |


|  | Q1 | Q2 | Q3 | Q4 |
| :--- | ---: | ---: | ---: | ---: |
| Beginning Cash Balance | $\$ 20$ | $\$ 60$ | $\$ 10$ | $\$ 10$ |
| Net cash inflow | 40 | -110 | 55 | -15 |
| New short-term borrowing | - | 60 | - | 15.4 |
| Interest on short-term borrowing (5\%) | - | - | 3 | 0.4 |
| Short-term borrowing repaid | - | - | 52 | - |
| Ending cash balance | $\$ 60$ | $\$ 10$ | $\$ 10$ | $\$ 10$ |
| Minimum cash balance | -10 | -10 | -10 | -10 |
| Cumulative surplus (deficit) | $\$ 50$ | 0 | 0 | 0 |
| Beginning short-term debt | 0 | 0 | 60 | 8 |
| Change in short-term debt | 0 | 60 | -52 | 15.4 |
| Ending short-term debt | $\$ 0$ | $\$ 60$ | $\$ 8$ | $\$ 23.4$ |

- Ending short-term debt is equal to cumulative deficit for the entire year, $\$ 20$ mill., plus interest paid during the year, $\$ 3+.4=\$ 3.4$, for a total of $\$ 23.4$
- In this plan, we ignored:
- the tax deductibility of interest paid
- Interest earnings on cash surplus
- This plan illustrates that financing the firm's short term needs will cost about $\$ 3.4$ million in interest before taxes for the year.


## Float and Cash Management

- The basic objective in cash management is to keep the investment in cash as low as possible while still operating the firm's activities efficiently and effectively.
- Collect early and pay late.
- Firm must invest temporarily idle cash in short term marketable securities, having lower default risk and most are highly liquid
- Reasons of holding Cash
- Speculative Motive - the need to hold cash to take advantage of additional investment opportunities, such as bargain purchases, attractive interest rates and favorable exchange rates fluctuations.
- Reserve borrowing utility and Marketable securities
- Transaction Motive - the need to hold cash to satisfy normal disbursement and collection activities associated with a firm's ongoing operations.
- Precautionary Motive - the need to hold cash as a safety margin to act as a financial reserve.
- Benefit of Holding Cash
- The opportunity cost of excessive cash is the interest income that could be earned in the next best use.
- The firm holds excessive cash to provide liquidity necessary for transaction needs.
- In case of cash-out situation, the firm may have to raise cash on a short-term basis by borrowing or selling current assets.


## FLOAT AND CASH MANAGEMENT

- The difference between bank cash and book cash, representing the net effect of cheques in the process of clearing is called float.
- Cheques written by a firm generate disbursement float, causing a decrease in the firm's book balance but no change in its available balance.
- Gm Inc. has $\$ 100,000$ on deposit with its bank.
- On June 8, it buys some raw material and pays with a cheque for $\$ 100,000$ reducing the book balance by $\$ 100,000$ immediately.
- GM's bank is unaware of this activity until the cheque is presented, say June 14 and the firm's available balance is greater than its book balance by $\$ 100,000$.
- Before June 8 , the GM's float is zero:

$$
\begin{aligned}
\text { Float } & =\text { Firm's available balance }- \text { Firm's book balance } \\
& =\$ 100,000-100,000=0
\end{aligned}
$$

- Form June 8 to June 14 , the position is:

$$
\begin{aligned}
\text { Disbursement Float } & =\text { Firm's available balance }- \text { Firm's book balance } \\
& =\$ 100,000-0=100,000
\end{aligned}
$$

- Cheques collected by the firm create collection float, which increases book balances but does not immediately change available balance.
- In general, the firm's payment activities generate disbursement float and its collection activities generate collection float.
- The net effect, the sum of both floats is the net float, equal to the overall difference between the firm's available balance and its book balance
- If net float is positive, then the firm's disbursement float exceeds its collection float and its available balance exceeds its book balance, and vice versa.
- Suppose you have $\$ 5,000$ on deposit.
- One day, you write a cheque for $\$ 1,000$ and deposit $\$ 2,000$.
- What are your disbursement, collection and net floats?
- After writing the cheque, you show a balance of $\$ 4,000$ on your books, but bank shows $\$ 5,000$ while the cheque is clearing. So you have a disbursement float of $\$ 1,000$
- Depositing $\$ 2,000$ increases the book balance to $\$ 6,000$, but the available balance remains the same until the cheque clears. So collection float is $-\$ 2,000$.

$$
\text { Net float }=\$ 1000+(-2,000)=-\$ 1,000
$$

- Cash Collection
- Speed up collection
- Reduce lag between the time customers pay their bills and time cash becomes available.
- Cash Disbursement
- Control payments
- Minimize firm's costs associated with making payments
- Total collection or disbursement time can be broken into three parts:
- Mailing time - the part of collection and disbursement process during which cheques are trapped in the postal system
- Processing delay - the time it takes the receiver of a cheque to process the payment and deposit in a bank for collection
- Availability delay - the time required to clear a cheque through the banking system
- Electronic Data Interchange (EDI)
- Electronically transfer financial information and funds between parties, thereby eliminating paper invoices, paper cheques, mailing and handling
- Length of time required to initiate and complete a business transaction is shortened
considerably, and float is sharply reduced or eliminated.


## Cash Collection



## Cash Collection

- Collection for payments through cheques
- Having mailed all cheques to one location
- Have different collection points to reduce mailing time
- Outsource the collection process
- Preauthorized payment system
- Lockboxes



## Cash Concentration

- Cash collections of firm having many collection points may end up in different banks and bank accounts. The process of moving these amounts to main account is called cash concentration.
- By routinely pooling its cash, the firm reduces the number of accounts that must be tracked.
- Normally, firms use concentration banks for this purpose.



## Cash Disbursement

- The goal in managing disbursement float is to slow down the disbursements as much as possible.
- The firm may adopt some strategies to increase mail float, processing float and availability float on the cheques it writes.
- These strategies may include:
- Write check on a distant bank.
- Hold payment for several days after postmarked in office.
- Call supplier firm to verify statement accuracy for large amounts.
- Mail from distant post office.
- Mail from post office that requires a great deal of handling



## Ethical and Legal Questions

- The financial managers must always work with collected company cash balances and not with the company's book balance, which reflects checks that have been deposited but not collected.
- If you are borrowing the bank's money without their knowledge, you are raising serious ethical and legal questions.


## Zero-balance account

- A disbursement account in which the firm maintains a zero balance transferring funds in from a master account only as needed to cover cheques presented for payment.


## No zero-balance accounts



Two zero-balance accounts


- Controlled disbursement accounts
- A disbursement account to which the firm transfers an amount that is sufficient to cover demands for payment.


## Investing Idle Cash

A firm with surplus cash can park it in the money market.

- Most large firms manage their own short-term financial assets, transacting through banks and dealers.
- Some large firms and many small ones use money market mutual funds (funds that invest in short term financial assets for some fee).
Firms have surplus cash mostly for two reasons:
- Seasonal or Cyclical Activities
- Planned Expenditures

- Time 1: A surplus cash position exists. Seasonal demand for current assets is low. The surplus is invested in short-term marketable securities.
- Time 2: deficit cash flow exists. Seasonal demand for assets is high. The financial deficit is financed by the selling of marketable securities and bank borrowing.


## Credits and Receivables

- The obvious reason for granting credit to the customers is to stimulate sales.
- The costs associated with granting assets are not trivial.
- Chance that customer will not pay
- Cost of carrying the receivables
- Credit policy decisions involve a trade-off between the benefits of increased sales and costs of granting credit.



## Components of Credit Policy

- Terms of sale

Conditions under which a firm sells its goods and services for cash or credit

- Credit Analysis

The process of determining the probability that customers will or will not pay.

- Collection Policy

Procedures followed by a firm in collecting accounts receivable

- The terms of sale of composed of
- Credit Period
- Cash Discounts and discount period
- Credit Instruments
- For example, the terms of $2 / 10$, net 30 may be quoted for a certain customer.
- This means that the customer has 30 days from invoice date to pay the full amount.
- If payment is made within 10 days, a $2 \%$ discount can be taken
- A buyer places an order for $\$ 1,000$ under the terms $2 / 10$, net 60 . He has the option of - paying $\$ 1,000 \times(1-.02)=\$ 980$ in 10 days, or
- paying the full $\$ 1,000$ in 60 days.
- In general, the credit terms are interpreted as:
(take this discount off the invoice price) / (if you pay in this many days), (else pay the full amount in this many days)


## CREDITS AND RECEIVABLES

## Credit Period

- The basic length of time for which the credit is granted.
- If a cash discount is offered, then credit period has two components
- Net credit period - length of time customer has to pay
- Cash discount period - time during which discount is available
- Invoice date is the shipping date or billing date of invoice, a written account of merchandise shipped to the buyer.
- Two most important factors influencing the length of credit period are buyer's inventory period and operating cycle. The shorter these are, the shorter the credit period will be
- We know that operating cycle has two components
- Inventory period - Receivables period
- By extending credit we finance a portion of our buyer's operating cycle and shorten the buyer's cash cycle.
- If our credit period exceeds the buyer's inventory period, we are not only financing buyer's inventory purchases but part of the buyer's receivables as well.
- If our credit period exceeds buyer's operating cycle, then we are providing financing for customer's business beyond the immediate purchase and sale of our merchandise.
- Buyer has a loan from us even after the merchandise is resold
- A number of other factors which influence the credit period are:
- Perish ability and collateral value
- Perishable items have rapid turnover and low collateral value; thus credit periods for these goods are shorter.
- Consumer demand
- Well established products have rapid turnover. While newer or slow moving products will require longer credit periods.
- Cost, profitability and standardization
- Inexpensive and standardized products have lower markups and higher turnover rates, leading to shorter credit period.
- Credit risk
- Higher credit risk of the buyer will lead to shorter credit period
- The size of the account
- For smaller accounts, credit periods may be shorter as they are more costly to manage
- Product market competition
- In highly competitive market, longer credit periods may be offered
- Customer type
- On case-to-case basis


## Cash discounts

- A discount given to induce prompt payment. It is also called sales discount
- This will have the effect of reducing the amount of credit offered, and firm must trade this off against cash discount.
- Buyer only credit after the discount expires
- This is a way to charging higher prices on credit sales
- An example would be " $3 / 10$ net 30 "
- The customer can take a $3 \%$ discount if he pays within 10 days.
- In any event, he must pay within 30 days.
- A firm offering credit terms of $3 / 10$ net 30 is essentially offering their customers a 20 -day loan.
- To see this, consider a firm that makes a $\$ 1,000$ sale on day 0
- Some customers will pay on day 10 and take the discount.

- Other customers will pay on day 30 and forgo the discount.

- A customer that forgoes the $3 \%$ discount to pay on day 30 is borrowing $\$ 970$ for 20 days and paying $\$ 30$ interest:
$\$ 970+\frac{\$ 1,000}{(1+\mathrm{r})^{\frac{20}{365}}}$
$(1+\mathrm{r})^{\frac{20}{365}}=\frac{\$ 1,000}{\$ 970}$
$\mathrm{r}=\left[\frac{\$ 1,000}{\$ 970}\right]^{\frac{365}{20}}-1$
$\mathrm{r}=1.7435-1$
$\mathrm{r}=0.7435$ or $74.35 \%$


## Credit Instrument

- It is a basic evidence of indebtness
- Most credit is offered on open account - the invoice is the only credit instrument.
- Promissory notes are IOUs that are signed after the delivery of goods
- Commercial drafts call for a customer to pay a specific amount by a specific date. The draft is sent to the customer's bank, when the customer signs the draft, the goods are sent.
- Banker's acceptances allow a bank to substitute its creditworthiness for the customer, for a fee.
- Conditional sales contracts let the seller retain legal ownership of the goods until the customer has completed payment.


## Optimal Credit Policy

- The optimal amount is determined by the point at which the incremental cash flows from increased sales are exactly equal to the incremental costs of carrying the increased investment in accounts receivable.
- The carrying costs associated with granting credit are of three types
- The required return on receivables
- The losses from bad debts
- The cost of managing credit and credit collection - expense associated with running the credit department.
- For a restrictive credit policy, all above costs will be low.
- Shortage of credit will be an opportunity cost of extra potential profit from credit sales that is low because credit is refused.
- This forgone benefit comes from
- Increase in quantity sold
- Higher prices
- Carrying costs are the cash flows that must be incurred when credit is granted. They are positively related to the amount of credit extended.
- Opportunity costs are the lost sales from refusing credit. These costs go down when credit is granted.
- The sum of carrying costs and opportunity costs is called the credit cost curve.

- Trade Credit is more likely to be granted if:
- The selling firm has a cost advantage over other lenders.
- The selling firm can engage in price discrimination.
- The selling firm can obtain favorable tax treatment.
- The selling firm has no established reputation for quality products or services.
- The selling firm perceives a long-term strategic relationship.
- The optimal credit policy depends on the characteristics of particular firms.
- Excess capacity
- Low variable operating costs
- Repeat customers


## Credit Analysis

- Credit analysis refers to the process of deciding whether or not to extend credit to a particular customer
- Gather relevant information
- Determine creditworthiness
- Credit Information
- Financial statements
- Credit reports on customer's payment history with other firms
- Banks
- Customer's payment history with the firm
- Credit Evaluation:
- The traditional 5 C's of credit:
- Character - Willingness to pay
- Capacity - Ability to pay
- Capital - Financial reserves
- Collateral - Pledged assets
- Conditions - Relevant economic conditions
- Credit scoring refers to the process of calculating a numerical rating for a customer based on information collected; credit is then granted or refused based on the result.


## Collection Policy

- Collection refers to obtaining payment on past-due accounts.
- Collection Policy is composed of:
- The firm's willingness to extend credit as reflected in the firm's investment in receivables.
- Collection Effort
- To keep track of the payments by the customers, the firm focuses on its Average Collection Period (ACP)
- Alternatively, Aging Schedule can also be used to monitor the receivables, which classifies credit accounts by age.
- Suppose a firm has $\$ 100,000$ in receivables. Some of the accounts are only a few days old, but others have been outstanding for quite some time.
- Its aging schedule may look like this:

| Age of Account | Amount | \% of Total Value of Accounts <br> Rec. |
| :---: | :---: | :---: |
| $0-10$ days | $\$ 50,000$ | $50 \%$ |
| $11-60$ days | 25,000 | $25 \%$ |
| $61-80$ days | 20,000 | $20 \%$ |
| Over 80 days | 5,000 | $5 \%$ |
|  | $\$ 100,000$ | $100 \%$ |

- If this firm has a credit period of 60 days, then $25 \%$ of its accounts are late.
- The seriousness of the case depends on the nature of firm's collections and customers.
- The longer an account has been unpaid, the less likely it is to be paid.
- Percentages on aging schedule keep changing for the firms with seasonal sales.
- Collection Effort
- Most firms follow a protocol for customers that are past due:
- Send a delinquency letter.
- Make a telephone call to the customer.
- Employ a collection agency.
- Take legal action against the customer.
- Firm may refuse to grant additional credit to customers until arrearages are cleared up.


## INVENTORY MANAGEMENT

- We know that firm's operating cycle is made up of its inventory period and its receivables period.
- Both credit policy and inventory policy are used to derive sales, and thus must be coordinated to ensure the process of acquiring \& selling inventory and collecting on sale proceeds smoothly.
- Inventory Types
- Raw Material - starting point in production process
- Work-in-Progress - size of this inventory depends on the length of production process
- Finished Goods
- One company's raw material can be another's finished good.
- Steel Sheets and Automobile Manufacturing
- Various types of inventory can be quite different in terms of their liquidity
- Commodity-like or standardized raw materials are easily converted to cash
- Work-in-progress can be quite illiquid
- Demand of an inventory item that becomes a part of another item is termed as Derived or Dependent Demand
- The firm's demand for finished goods not derived from demand for other inventory items
- Inventory Costs
- Carrying costs include all direct and opportunity costs of keeping inventory on hand
- Storage and tracking costs
- Insurance and taxes
- Losses due to obsolescence, deterioration, or theft
- Opportunity cost of capital for the invested amount
- Shortage costs are associated with having inadequate inventory on hand
- Restocking costs - costs of placing an order with suppliers or the cost of setting up a production run
- Safety reserve costs - opportunity losses from having inadequate inventory e.g. lost sales and goodwill
- A trade-off
- Carrying costs increase with inventory levels and shortage or restocking costs decline with inventory levels
- The goal of inventory management is to minimize the sum of these two costs.


## ABC Approach

- Basic idea in this approach of inventory management is to divide the inventory into three (or more) groups.
- A small portion of inventory in terms of quantity might represent a large portion in terms of inventory value.
- A production process involving expensive, high-tech components as well as relatively inexpensive basic materials.



## Economic Order Quantity Model

- Economic order quantity (EOQ) model is the best known approach to explicitly establish an optimal inventory level.


## Cost of holding

inventory (\$)


Optimal size of inventory order

- Actual cost of inventory is not included as the total amount of inventory the firm needs in a given year is dictated by sales
- Here we are trying to determine the order size the firm should use when it restocks its inventory
- Assuming that the firm's inventory is sold off at a steady rate until it hits zero. At that point firm restocks back to some optimal level
- Suppose a firm starts off today with 3,600 units of a particular item in inventory. Annual sales
of this item are 46,800 units (about 900 per week).
- With this pace of weekly sales, all the available inventory will be sold after four weeks, and the firms will restock by ordering (or manufacturing) another 3,600 units and start over.
- This selling and restocking process produces a saw-tooth pattern for inventory holdings:

- We have seen that carrying costs are assumed to be directly proportional to inventory levels.
- If we let CC be the carrying cost per unit per year, the firm's total carrying costs will be:

$$
\begin{aligned}
\text { Total carrying costs } & =\text { Avg. inventory } \times \text { carrying costs per unit } \\
& =(\mathrm{Q} / 2) \times \mathrm{CC}
\end{aligned}
$$

- For carrying costs of $\$ 0.75$ per unit per year total carrying costs will be $1,800 \mathrm{X} 0.75=\$ 1,350$ per year.
- In case of shortage costs we will focus only on restocking costs, assuming that firm never runs short of inventory.
- Restocking costs are normally assumed to be fixed.
- Now, let 'T' be the firm's total unit sales per year. If firm orders q units each time, it will need to place a total of T/Q orders.
- In our example this number would be $46,800 / 3,600=13$ orders per year
- If fixed cost per order is F , then total restocking costs for the year will be:

$$
\begin{aligned}
\text { Total restocking costs } & =\text { Fixed costs per order } \times \text { Number of orders } \\
& =\mathrm{F} \times(\mathrm{T} / \mathrm{Q})
\end{aligned}
$$

- In our example order costs might be $\$ 50$ per order, so total restocking costs for 13 orders will be $\$ 50 \times 13=\$ 650$ per year.

$$
\begin{aligned}
\text { Total costs } & =\text { carrying costs }+ \text { restocking costs } \\
& =(\mathrm{Q} / 2) \times \mathrm{CC}+\mathrm{F} \times(\mathrm{T} / \mathrm{Q})
\end{aligned}
$$

- To find the value of Q , the restocking quantity, that minimizes this cost, we calculate the total costs for some different values of Q .
- We had carrying costs (CC) of $\$ 0.75$ per unit per year, fixed costs ( F ) of $\$ 50$ per order and total unit sales (T) of 46,800 units.

| Restocking Quantity <br> $(\mathbf{Q})$ | Total carrying costs <br> $(\mathbf{Q} / \mathbf{2} \times \mathbf{C C})$ | Restocking Costs <br> $(\mathbf{F} \times \mathbf{T} / \mathbf{Q})$ | Total Costs |
| :---: | :---: | :---: | :---: |
| 500 | $\$ 187.5$ | $\$ 4,680.0$ | $\$ 4,867.5$ |
| 1,000 | 375.0 | $2,340.0$ | $2,715.0$ |
| 1,500 | 562.5 | $1,560.0$ | $2,122.5$ |
| 2,000 | 750.0 | $1,170.0$ | $1,920.0$ |
| 2,500 | 937.5 | 936.0 | $\mathbf{1 , 8 7 3 . 5}$ |
| 3,000 | $1,125.0$ | 780.0 | $1,905.0$ |
| 3,500 | $1,312.5$ | 668.6 | $1,981.1$ |

- To find the precise cost-minimization quantity we take a look at the basic EOQ Model diagram.


## Cost of holding <br> inventory (\$) <br> 

- We can find the minimum point just by equating carrying and restocking costs and solving for Q*

> Carrying costs $=$ Restocking Costs $\left(\mathrm{Q}^{*} / 2\right) \times \mathrm{CC}=\mathrm{F} \times\left(\mathrm{T} / \mathrm{Q}^{*}\right)$

- Rearranging,

$$
\begin{array}{ll} 
& \left(\mathrm{Q}^{*}\right)^{2}=2 \mathrm{~T} \times \mathrm{F} / \mathrm{CC} \\
\text { or } & \mathrm{Q}^{*}=(2 \mathrm{~T} \times \mathrm{F} / \mathrm{CC})^{1 / 2}
\end{array}
$$

- This reorder quantity which minimizes the total inventory cost is called the Economic order quantity (EOQ).
- For our example, the EOQ is:

$$
\begin{aligned}
& \mathrm{Q}^{*}=(2 \mathrm{~T} \times \mathrm{F} / \mathrm{CC})^{1 / 2} \\
& \mathrm{Q}^{*}=[(2 \times 46,800) \times \$ 50 / 0.75]^{1 / 2} \\
& \mathrm{Q}^{*}=2,498 \text { units }
\end{aligned}
$$

## Carrying Costs

- R Shoes begins each period with 100 pairs of hiking boots in stock, which is depleted each period and reordered. If carrying cost per pair of boots per year is $\$ 3$, what are the total
carrying costs for hiking boots?
- Average inventory is $100 / 2=50$ items. At an annual cost of $\$ 3$ per item, total carrying costs are $\$ 150$.


## Restocking Costs

- Suppose R Shoes sells a total of 600 pairs a year. How many times per year does the company restock?
- If the restocking cost is $\$ 20$ per order, what are total restocking costs?
- R Shoes restocks $600 / 100=6$ times per year or about every two months.
- The restocking costs will be 6 orders $\mathrm{x} \$ 20$ per order $=\$ 120$.


## The EOQ

- What size orders shout R Shoes place to minimize costs? How often will the company restock? What are the total carrying and restocking costs? What are the total costs?
- Here $\mathrm{T}=600, \mathrm{~F}=\$ 20, \mathrm{CC}=\$ 3$, so EOQ for R shoes is

$$
\begin{aligned}
& \mathrm{EOQ}=(2 \mathrm{~T} \times \mathrm{F} / \mathrm{CC})^{1 / 2} \\
& \mathrm{EOQ}=[(2 \times 600) \times \$ 20 / 3]^{1 / 2} \\
& \mathrm{EOQ}=89.44 \text { units }
\end{aligned}
$$

- Since $R$ shoes, sells 600 pairs per year, it will stock / $89.44=6.71$ times
- The total restocking costs will be $\$ 20 \times 6.71=\$ 134.16$.
- Average inventory will be $89.44 / 2=44.72$
- The carrying costs will be $\$ 3 \times 44.72 \$ 134.16$, the same as restocking costs
- Total costs are thus $\$ 268.33$
- In reality, firms wish to reorder before its inventory runs down to zero
- Firms minimize the risk of stock-out
- Firm covers the lead time between order and delivery
- Safety stock
- The minimum level of inventory that a firm keeps on hand, inventories are reordered the level of inventory falls to the safety stock level.


## Inventory (units)



Time

- Reorder points
- These are the times at which the firm will actually place its inventory orders even before reaching critical level.


