Only students of Section: Nabeela Wali shall solve this Assignment.

Assignment # 01 MTH603 (Spring 2024)

Marks: 10
Due Date: May 6, 2024

DON'T MISS THESE: Important instructions before attempting the solution and submission of this assignment:

- **Lectures 3-10 are encompassed in Assignment 1.**
- **Only students in Section: Nabeela Wali shall complete this Assignment.**
- **Students in other teacher's section will be completing a different assignment and are strictly prohibited from solving this one.**
- **Submitting a copied assignment or irrelevant assignment will result in a zero grade.**
- **Assignment 1 is due on May 6, 2024.**
- **Properly Upload the solution of this assignment in MS Word format on LMS as per the previous practice.**

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Question 1 **5 Marks**

Find a root of the equation $x^3 - 3x - 5 = 0$, in the interval (2,3) using Bisection Method after three Iterations. Note: Accuracy up to four decimal places is required.

Solution:

Here $x^3 - 3x - 5 = 0$

Let $f(x) = x^3 - 3x - 5$

1^{st} iteration :

Here $f(2) = -3 < 0$ and $f(3) = 13 > 0$

$\therefore$ Now, Root lies between 2 and 3

$$x_0 = \frac{2+3}{2} = 2.5$$

$$f(x_0) = f(2.5) = 2.5^3 - 3 \cdot 2.5 - 5 = 3.125 > 0$$

2^{nd} iteration :

Here $f(2) = -3 < 0$ and $f(2.5) = 3.125 > 0$

$\therefore$ Now, Root lies between 2 and 2.5

$$x_1 = \frac{2+2.5}{2} = 2.25$$

$$f(x_1) = f(2.25) = 2.25^3 - 3 \cdot 2.25 - 5 = -0.3594 < 0$$

3^{rd} iteration :

Here $f(2.25) = -0.3594 < 0$ and $f(2.5) = 3.125 > 0$

$\therefore$ Now, Root lies between 2.25 and 2.5

$$x_2 = \frac{2.25+2.5}{2} = 2.375$$

$$f(x_2) = f(2.375) = 2.375^3 - 3 \cdot 2.375 - 5 = 1.2715 > 0$$

Hence, the approximate root is 2.375 after 3rd iteration.

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Question 2 **5 Marks**

Find a root of the following equation in the interval (0,1) using Newton-Raphson Method after three iterations

$$xe^x - \cos x = 0$$

Take Initial value 0.5.

Note: Accuracy up to four decimal places is required. Here is a transcendental equation all the calculation should be done in the radians mode.

Solution:

As we know Newton –Raphson Formula is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

Here $xe^x - \cos(x) = 0$

Let $f(x) = xe^x - \cos(x)$

$\therefore f'(x) = e^x + xe^x + \sin(x)$

Here

x	0	1
$f(x)$	-1	2.178

Here $f(0) = -1 < 0$ and $f(1) = 2.178 > 0$

$\therefore$ Root lies between 0 and 1

$$x_0 = \frac{0+1}{2} = 0.5$$

$$x_0 = 0.5$$

1^{st} iteration :

$$f(x_0) = f(0.5) = 0.5e^{0.5} - \cos(0.5) = -0.0532$$

$$f'(x_0) = f'(0.5) = e^{0.5} + 0.5e^{0.5} + \sin(0.5) = 2.9525$$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = 0.5 - \frac{-0.0532}{2.9525}$$

$$x_1 = 0.518$$

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2^{nd} iteration :

$$f(x_1) = f(0.518) = 0.518e^{0.518} - \cos(0.518) = 0.0008$$

$$f'(x_1) = f'(0.518) = e^{0.518} + 0.518e^{0.518} + \sin(0.518) = 3.0435$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$x_2 = 0.518 - \frac{0.0008}{3.0435}$$

$$x_2 = 0.5178$$

3^{rd} iteration :

$$f(x_2) = f(0.5178) = 0.5178e^{0.5178} - \cos(0.5178) = 0$$

$$f'(x_2) = f'(0.5178) = e^{0.5178} + 0.5178e^{0.5178} + \sin(0.5178) = 3.0421$$

$$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$$

$$x_3 = 0.5178 - \frac{0}{3.0421}$$

$$x_3 = 0.5178$$

Hence , the root is 0.5178.