MTH603 Assignment 2 Solution Spring 2021

Question No 1

Solution

Let x=1.5 appear at the beginning of the table so we use the forward difference formula to find its derivative.

Here h=0.5 (Difference of x values)

x	$f(x)$	$\Delta f(x)$	$\Delta^2 f(x)$	$\Delta^3 f(x)$	$\Delta^4 f(x)$	$\Delta^5 f(x)$
1.5	3.375					
2.0	7.000	3.625				
2.5	13.625	6.625	3.000			
3.0	24.000	10.375	3.750	0.750		
3.5	38.875	14.875	4.500	0.750	0	
4.0	59.000	20.125	5.250	0.750	0	0

First Derivative

$$Df(x) = \frac{1}{h}\left[\Delta f(x) - \frac{\Delta^2 f(x)}{2} + \frac{\Delta^3 f(x)}{3} - \frac{\Delta^4 f(x)}{4}\right]$$

$$f'(1.5) = \frac{1}{5}\left[3.625 - \frac{3}{2} + \frac{0.750}{3} - \frac{0}{4}\right]$$

$$f'(1.5) = 2\left[3.625 - 1.5 + 0.25 - 0\right]$$

$$f'(1.5) = 4.75$$

2nd Derivative

Using the formula $D^2f(x)$

$$D^2 f(x) = \frac{1}{h^2}\left[\Delta^2 f(x) - \Delta^3 f(x) + \frac{11}{12}\Delta^4 f(x) - \frac{5}{6}\Delta^5 f(x)\right]$$

$$f''(1.5) = \frac{1}{(0.5)^2}\left[3.000 - 0.750 + \frac{11}{12}(0) - \frac{5}{6}(0)\right]$$

$$f''(1.5) = 4\left[3.000 - 0.750 + 0 - 0\right]$$

$$f''(1.5) = 9$$

Question No 2

Solution

The Farward defferences are tabulated as:

x	$f(x)$	Δy	$\Delta^2 y$	$\Delta^3 y$
1	3.49			
1.4	4.82	1.33		
1.8	5.96	1.14	- 0.19	
2.2	6.5	0.54	- 0.6	- 0.41

Here h = 0.4

Farward defferences Interpolation formula:

$$y_x = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \ldots\ldots + \frac{p(p-1)(p-n+1)}{n!}\Delta^n y_0 + error$$

Let $y_{1.6}$ be the value of y when x=1.6 then

$$p = \frac{x - x_0}{h}$$

$$f(1.6) = y_{1.6} = 3.49 + (1.5)(1.33) + \frac{1.5(1.5-1)}{2!}(-0.19) + \frac{1.5(1.5-1)(1.5-2)}{3!}(-0.41)$$

$$f(1.6) = 3.49 + 1.995 + 0.0712 + 0.0256$$

$$f(1.6) = 5.4394$$

THE END