Virtual University of Pakistan

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MTH603

Numerical Analysis

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 Note: Use Table Of Content to view the Topics, In PDF(Portable Document Format) format , you can check Bookmarks menu

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Introduction To Numerical Analysis

Course Content:

Number systems, Errors in computation, Methods of solving non-linear equations, Solution of linear system of equations and matrix inversion, Eigen value problems, power method, Jacobi's method, Different techniques of interpolation, Numerical differentiation and integration, Numerical integration formulas, different methods of solving ordinary differential equations.

Each Topic One Solved Question with Steps:Compiled from Past assignment

Preliminary material:

Representation of numbers

Binary Number System (Base 2)=0,1

Used in Computer Internal Operations and Calculation

Octal Number System (Base 8)=0,1,2,3,4,5,6,7

ERROR

Types of error are,
- Inherent errors,
- Local round-off errors
- Local truncation errors

Decimal Number System (Base 10)=0,1,2,3,4,5,6,7,8,9

World Wide used and the most common system

Hexa Number System (Base 16)=0,1,2,3,4,5,6,78,9,A,B,C,D,E,F

Used in Processor Register Addressing

Errors in computations

The initial approximation

It may be found by two methods either by graphical method or
Analytical method
Graphical method

Non-Linear Equations

Bisection Method

Procedure in Detail:

Step,Take two Initial Approximation such that f(x1).F(x2)<0.Means both must have opposite signs.Take their mean by x3=(x1+x2)/2

Next Take two element from which 1 will be x3 and another from x1 or x2 such that both x3 and the other element should have opposite sign.

Repeat the above process to the required numbers of iterations.

Question 1

Find the root of the equation given below by bisection method.

$$x^3 - x^2 + x - 7 = 0$$

(Note: accuracy up to three decimal places is required.)   **Marks: 10**

SOLUTION

Let

$$f(x) = x^3 - x^2 + x - 7$$

Now $f(1) = (1)^3 - (1)^2 + 1 - 7 = -6 < 0$

$f(2) = (2)^3 - (2)^2 + 2 - 7 = -1 < 0$

and $f(3) = (3)^3 - (3)^2 + 3 - 7 = 14 > 0$

As $f(2)f(3) = -14 < 0$

Therefore a real root lies between 2 and 3

Iteration 1

Let

$x_0 = 2 \, and \, x_1 = 3$

then

$$x_2 = \left(\frac{x_0 + x_1}{2}\right)$$

$$= \left(\frac{2+3}{2}\right) = 2.5$$

Now $f(2.5) = (2.5)^3 - (2.5)^2 + 2.5 - 7 = 4.875 > 0$

As $f(2)f(2.5) = -1(4.875) = -4.875 < 0$

Therefore a real root lies between 2 and 2.5

Iteration 2

$x_0 = 2 \, and \, x_2 = 2.5$

then

$$x_3 = \left(\frac{x_0 + x_2}{2}\right)$$

$$= \left(\frac{2+2.5}{2}\right) = 2.25$$

Now $f(2.25) = (2.25)^3 - (2.25)^2 + 2.25 - 7 = 1.578125 > 0$

As $f(2)f(2.25) = -1(1.578125) = -1.578125 < 0$

Therefore a real root lies between 2 and 2.25

Iteration 3

$x_0 = 2 \, and \, x_3 = 2.25$

then

$$x_4 = (\frac{x_0 + x_3}{2})$$

$$= (\frac{2 + 2.25}{2}) = 2.125$$

Now $f(2.125) = (2.125)^3 - (2.125)^2 + 2.125 - 7 = 0.205078 > 0$

As $f(2)f(2.125) = -1(0.205078) = -0.205078 < 0$

Therefore a real root lies between $2 \, and \, 2.125$

Iteration 4

$x_0 = 2 \, and \, x_4 = 2.125$

then

$$x_5 = (\frac{x_0 + x_4}{2})$$

$$= (\frac{2 + 2.125}{2}) = 2.0625$$

Now $f(2.0625) = (2.0625)^3 - (2.0625)^2 + 2.0625 - 7 = -0.417725 < 0$

As $f(2.0625)f(2.125) = -0.417725(0.205078) = -0.085666 < 0$

Therefore a real root lies between $2.0625 \, and \, 2.125$

Iteration 5

$x_5 = 2.0625 \, and \, x_4 = 2.125$

then

$$x_6 = (\frac{x_5 + x_4}{2})$$

$$= (\frac{2.0625 + 2.125}{2}) = 2.09375$$

Now $f(2.09375) = (2.09375)^3 - (2.09375)^2 + 2.09375 - 7 = -0.111481 < 0$

As $f(2.09375)f(2.125) = -0.111481(0.205078) = -0.022862 < 0$

Therefore a real root lies between $2.09375 \, and \, 2.125$

Iteration 6

$x_6 = 2.09375 \, and \, x_4 = 2.125$

then

$$x_7 = (\frac{x_6 + x_4}{2})$$

$$= (\frac{2.09375 + 2.125}{2}) = 2.109375$$

Now $f(2.109375) = (2.109375)^3 - (2.109375)^2 + 2.109375 - 7 = 0.0455 > 0$

As $f(2.09375)f(2.109375) = -0.111481(0.0455) = -0.00507 < 0$

Therefore a real root lies between $2.09375 \, and \, 2.109375$

Iteration 7

$x_6 = 2.09375 \, and \, x_7 = 2.109375$

then

$x_8 = (\dfrac{x_6 + x_7}{2})$

$= (\dfrac{2.09375 + 2.109375}{2}) = 2.1015625$

Now $f(2.1015625) = (2.1015625)^3 - (2.1015625)^2 + 2.1015625 - 7 = -0.0333 < 0$

As $f(2.1015625) f(2.109375) = -0.0333(0.0455) = -0.01515 < 0$

Therefore a real root lies between $2.1015625 \, and \, 2.109375$

Iteration 8

$x_8 = 2.1015625 \, and \, x_7 = 2.109375$

then

$x_9 = (\dfrac{x_8 + x_7}{2})$

$= (\dfrac{2.1015625 + 2.109375}{2}) = 2.10546875$

Now $f(2.10546875) = (2.10546875)^3 - (2.10546875^2 + 2.10546875 - 7 = 0.006 > 0$

As $f(2.1015625) f(2.10546875) = -0.0333(0.006) = -0.0002 < 0$

Therefore a real root lies between $2.1015625 \, and \, 2.10546875$

Iteration 9

$x_8 = 2.1015625 \, and \, x_9 = 2.10546875$

then

$x_{10} = (\dfrac{x_8 + x_9}{2})$

$= (\dfrac{2.1015625 + 2.10546875}{2}) = 2.103515625$

Now $f(2.103515625) = (2.103515625)^3 - (2.103515625)^2 + 2.103515625 - 7 = -0.01367 < 0$

As $f(2.103515625) f(2.10546875) = -0.01367(0.006) = -0.00008 < 0$

Therefore a real root lies between $2.103515625 \, and \, 2.10546875$

Iteration 10

$x_{10} = 2.103515625 \, and \, x_9 = 2.10546875$

then

$x_{11} = (\dfrac{x_{10} + x_9}{2})$

$= (\dfrac{2.103515625 + 2.10546875}{2}) = 2.104492188$

Now $f(2.104492188) = (2.104492188)^3 - (2.104492188)^2 + 2.104492188 - 7 = -0.0038 < 0$

As $f(2.104492188) f(2.10546875) = -0.0038(0.006) = -0.0000228 < 0$

Therefore a real root lies between $2.104492188 \, and \, 2.10546875$

Iteration 11

$x_{11} = 2.104492188 \, and \, x_9 = 2.10546875$

then

$x_{12} = (\dfrac{x_{11} + x_9}{2})$

$= (\dfrac{2.104492188 + 2.10546875}{2}) = 2.104980469$

Now $f(2.104980469) = (2.104980469)^3 - (2.104980469)^2 + 2.104980469 - 7 = 0.0011 > 0$

As $f(2.104492188) f(2.104980469) = -0.0038(0.0011) = -0.00000418 < 0$

Therefore a real root lies between $2.104492188 \, and \, 2.104980469$

As the next root lies between $2.104492188 \, and \, 2.104980469$ and these roots are equal up to three decimal places.

So, the required root up to three decimal places is 2.104

<mark>Example From Handout at page # 4</mark>

Solve $x^3 - 9x + 1 = 0$ for the root between x=2 and x=4 by bisection method

Solution:

Here we are given the interval (2,4) so we need not to carry out intermediate value

property to locate initial approximation.

Here   f(x) $= x^3 - 9x + 1 = 0$

Now f(2)=-9 and f(4)=29.

Here f(2).f(4)<0 So the root lies between 2 and 4.

So, $x_o = 2, x_1 = 4$

and $x_2 = \dfrac{x_o + x_1}{2}$ =3 (This formula predicts next iteration)

Now f(3)=1,Here f(2).f(3)<0 So the root lies between 2 and 3.

Repeat above process to required number of iteration.

Step,Take two Initial Approximation such that f(x1).F(x2)<0.Means both must have opposite signs.Take their mean by x3=(x1+x2)/2

Next Take two element from which 1 will be x3 and another from x1 or x2 such that both x3 and the other element should have opposite sign.

Repeat the above process to the required numbers of iterations.

Regula Falsi Method

Formula for the Regula Falsi Method is $x3 = x2 - \dfrac{x2 - x1}{f(x2) - f(x1)} f(x2)$ or

$$x_{(n+1)} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n)$$

Steps: If the interval is given , check whether the root lies in between the interval or not by the principle that states if both number have opposite sign , then the root lies in between interval.

Find the next approximation with the help of the formula.

Question#2   **Marks 10**

Use Regula-Fasli method to compute the root of the equation $f(x) = \cos x - xe^x$ In the interval [0, 1] after third iteration.

Solution:

As $f(x) = \cos x - xe^x$

$f(0) = \cos 0 - 0e^0 = 1 \succ 0$

$f(1) = \cos 1 - 1e^1 = 0.5403 - 2.718 = -2.1779 \prec 0$

So root of the eq. lie between 0 *and* 1. *Let* $x_0 = 0$ *and* $x_1 = 1$

Thus $f(x_0) = 1$ *and* $f(x_1) = -2.1779$

The formula for finding the root of the function f(x) by Regula-Fasli is given by.

$$x_{(n+1)} = x_n - \frac{x_n - x_{n-1}}{f(x_n) - f(x_{n-1})} f(x_n)$$

For n=1 we have,

$$x_2 = x_1 - \frac{x_1 - x_o}{f(x_1) - f(x_o)} f(x_1) = 1 - \frac{1-0}{-2.17797 - 1}(-2.17797) = 1 - \frac{2.17797}{3.17797} = 1 - 0.68533 = 0.31467$$

$$f(x_2) = \cos(0.31467) - (0.31467)e^{0.31467} = 0.950899 - (0.31467)(1.369806) = 0.519863$$

Since $f(x_1)$ and $f(x_2)$ are of opposite signs, the root lies between x_1 and x_2. Therefore, for n=2 we have,

$$x_3 = x_2 - \frac{x_2 - x_1}{f(x_2) - f(x_1)} f(x_2) = 0.31467 - \frac{0.31467 - 1}{0.519863 - (-2.17797)}(0.519863)$$

$$= 0.31467 + \frac{0.68533}{2.697833}(0.519863) = 0.44673$$

$$f(x_3) = \cos(0.44673) - (0.44673)e^{0.44673} = 0.901864 - (0.44673)(1.563191) = 0.20353$$

Since $f(x_1)$ and $f(x_3)$ are of opposite signs, the root lies between x_1 and x_3. Therefore, for n=3 we have,

$$x_4 = x_3 - \frac{x_3 - x_1}{f(x_3) - f(x_1)} f(x_2) = 0.44673 - \frac{0.44673 - 1}{0.20353 - (-2.17797)}(0.20353)$$

$$= 0.44673 + \frac{0.54327}{2.3815}(0.20353) = 0.493159$$

The required root after 3rd iteration using Regula-Falsi method is 0.493159

Question 2

Use the Regula Falsi (method of false position) to solve the equation $x^3 - 4x - 9 = 0$

(Note: accuracy up to four decimal places is required)   **Marks: 10**

SOLUTION

Let

$f(x) = x^3 - 4x - 9$

Now

$f(0) = (0)^3 - 4(0) - 9 = 0 - 0 - 9 = -9 < 0$

$f(1) = (1)^3 - 4(1) - 9 = 1 - 4 - 9 = -12 < 0$

$f(2) = (2)^3 - 4(2) - 9 = 8 - 8 - 9 = -9 < 0$

$f(3) = (3)^3 - 4(3) - 9 = 27 - 12 - 9 = 6 > 0$

Now, Since $f(2)$ *and* $f(3)$ *are of opposite signs,*

therefore, the real root lies between 2 *and* 3

Now,

let $x_1 = 2$ *and* $x_2 = 3$

First iteration

$x_3 = x_2 - \dfrac{x_2 - x_1}{f(x_2) - f(x_1)} f(x_2)$

$= 3 - \dfrac{3 - 2}{6 - (-9)}(6)$

$= 3 - \dfrac{1}{15}(6)$

$= 3 - \dfrac{2}{5}$

$= 2.6$

$f(x_3) = f(2.6) = (2.6)^3 - 4(2.6) - 9$

$\qquad = -1.824$

Second iteration

$x_4 = x_3 - \dfrac{x_3 - x_2}{f(x_3) - f(x_2)} f(x_3)$

$= 2.6 - \dfrac{2.6 - 3}{-1.824 - 6}(-1.824)$

$= 2.69325$

$f(x_4) = f(2.69325) = (2.69325)^3 - 4(2.69325) - 9$

$\qquad = -0.23725$

Third iteration

$$x_5 = x_4 - \frac{x_4 - x_3}{f(x_4) - f(x_3)} f(x_4)$$

$$= 2.69325 - \frac{2.69325 - 2.6}{-0.23725 - (-1.824)}(-0.2061)$$

$$= 2.70536$$

$$f(x_5) = f(2.70536) = (2.70536)^3 - 4(2.70536) - 9$$

$$= -0.02098$$

Fourth iteration

$$x_6 = x_5 - \frac{x_5 - x_4}{f(x_5) - f(x_4)} f(x_5)$$

$$= 2.70536 - \frac{2.70536 - 2.69325}{-0.02098 - (-0.23725)}(-0.02098)$$

$$= 2.70653$$

$$f(x_6) = f(2.70653) = (2.70653)^3 - 4(2.70653) - 9$$

$$= 0.0000368$$

Fifth iteration

$$x_7 = x_6 - \frac{x_6 - x_5}{f(x_6) - f(x_5)} f(x_6)$$

$$= 2.70653 - \frac{2.70653 - 2.70536}{0.0000368 - (-0.02098)}(0.0000368)$$

$$= 2.70653$$

Hence,

The required root of the given equation correct to 4 decimal places is 2.7065

Method of Iterations

Question#1 **Marks 10**

Use the method of iterations to determine the real root of the equation $e^{-x} = 10x$ in the interval [0, 1], correct to four decimal places after four Iterations.

Solution:

Since $e^{-x} = 10x$. Let $f(x) = e^{-x} - 10x$. we see that $f(0) = 1$ and $f(1) = -9.6321$

As $e^{-x} = 10x$, we can easily find the value of x, thus

$x = \dfrac{e^{-x}}{10}$, Let $\phi(x) = \dfrac{e^{-x}}{10}$

By taking its derivative we get

$\phi'(x) = -\dfrac{e^{-x}}{10}$, we see that $|\phi'(x)| \prec 1$ for all values in $[0,1]$.

Therefore we can apply method of iterations to the given function.

Take any value within $[0,1]$.*Let*

$x_0 = 0.5$

$x_1 = \phi(0.5) = -\dfrac{e^{-0.5}}{10} = 0.0607$, $\qquad f(x_1) = -0.09516$

$x_2 = \phi(0.0607) = -\dfrac{e^{-0.0607}}{10} = 0.0914$, $\quad f(x_2) = 0.00869$

$x_3 = \phi(0.0914) = -\dfrac{e^{-0.0914}}{10} = 0.0941$, $\quad f(x_3) = -0.000790477 = -7.90477 x 10^{-4}$

$x_4 = \phi(0.0941) = -\dfrac{e^{-0.0941}}{10} = 0.0913$, $\quad f(x_4) = 0.0000275784 = 2.75784 x 10^{-5}$

$x_4 = 0.0913$

Example From the Handout at page #

Example

Find a real root of the equation $\cos x = 3x - 1$ correct to seven places of decimal.

Solution

Here it is a transcendental function and all the calculation must be done in the radians mode and value of pi should be 3.14

$f(x) = \cos x - 3x + 1$

$f(0) = \cos 0 - 3(0) + 1 = 1 > 0$

$f(\pi/2) = \cos(1.57) - 3(1.57) + 1 = 0.0007963 - 4.71 + 1 = -3.7092037 < 0$

so a real root lies between 0 *and* $\pi/2$

here $\phi(x) = \dfrac{1}{3}(\cos x + 1)$

we have $\phi'(x) = -\dfrac{1}{3}\sin x$

it is clearly less than 1 *as sin is a bounded function and it's values lies between* -1 *and* 1
hence iteration method can be applied
let $x_0 = 0.5$ *be the inital approximation then*

$x_1 = \phi(x_0) = \dfrac{1}{3}[\cos(0.5) + 1] = 0.6258608$

$x_2 = \phi(x_1) = \dfrac{1}{3}[\cos(0.6258608) + 1] = 0.6034863$

$x_3 = \phi(x_2) = \dfrac{1}{3}[\cos(0.6034863) + 1] = 0.6077873$

$x_4 = \phi(x_3) = \dfrac{1}{3}[\cos(0.6077873) + 1] = 0.6069711$

$x_5 = \phi(x_4) = \dfrac{1}{3}[\cos(0.6069711) + 1] = 0.6071264$

$x_6 = \phi(x_5) = \dfrac{1}{3}[\cos(0.6071264) + 1] = 0.6070969$

$x_7 = \phi(x_6) = \frac{1}{3}[\cos(0.6070969) + 1] = 0.6071025$

$x_8 = \phi(x_7) = \frac{1}{3}[\cos(0.6071025) + 1] = 0.6071014$

$x_9 = \phi(x_8) = \frac{1}{3}[\cos(0.6071014) + 1] = 0.6071016$

$x_{10} = \phi(x_9) = \frac{1}{2}[\cos(0.6071016) + 1] = 0.6071016$

Newton-Raphson Method

Procedural Detail:
Find the limit if not provided by starting from x=0 to ,…
Find two consecutive numbers for f(x) should have opposite sign.
Use the Newton Raphson Formula to find next approximation .

Question#3   **Marks 10**

Find the real root of the equation $x^3 - 3x - 5 = 0$ using Newton-Raphson method in the interval [2,3] after third iteration.

Solution:

$As\ f(x) = x^3 - 3x - 5$

$f(2) = 2^3 - 3.2 - 5 = -3 \prec 0$

$f(3) = 3^3 - 3.3 - 5 = 13 \succ 0$

$So\ root\ of\ the\ eq.\ will\ lie\ in\ [2,3]$

$now\ f'(x) = 3x^2 - 3\ and\ f''(x) = 6x$

$f'(2) = 3.2^2 - 3 = 9\ and\ f'(3) = 3.3^2 - 3 = 24$

$f''(2) = 6.2 = 12\ and\ f''(3) = 6.3 = 18$

$Since\ f(3)\ and\ f''(3)are\ of\ same\ sign.So\ we\ choose\ x_0 = 3$

$so\ by\ Newton's\ method\ we\ have$

$x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)} = 3 - \dfrac{13}{24} = 2.4583$

$f(x_1) = (2.4583)^3 - 3(2.4583) - 5 = 14.8561 - 7.3749 - 5 = 2.4812$

$f'(x_1) = f'(2.4583) = 3(2.4583)^2 - 3 = 18.1297 - 3 = 15.1297$

$x_2 = x_1 - \dfrac{f(x_1)}{f'(x_1)} = 2.4583 - \dfrac{2.4812}{15.1297} = 2.2943$

$f(x_2) = (2.2943)^3 - 3(2.2943) - 5 = 12.0767 - 6.8829 - 5 = 0.1938$

$f'(x_2) = f'(2.2943) = 3(2.2943)^2 - 3 = 15.7914 - 3 = 12.7914$

$x_3 = x_2 - \dfrac{f(x_2)}{f'(x_2)} = 2.2943 - \dfrac{0.1938}{12.7914} = 2.2791$

Example From the Handout at page #
Value of e=2.7182

Secant Method

Question 1

Do Four iterations of Secant method, with an accuracy of 3 decimal places to find the
.
root of

$$f(x) = x^3 - 3x + 1 = 0, \quad x_0 = 1, x_1 = 0.5$$ **Marks: 10**

Solution:

FORMULA OF SECANT METHOD

$$x_{n+1} = \frac{x_{n-1}(fx_n) - x_n f(x_n - 1)}{(fx_n) - f(x_n - 1)}$$

ITERATION 1

$n = 1$

$f(x_0) = f(1) = -1$

$f(x_1) = f(0.5) = -0.375$

$x_2 = \frac{x_o(fx_1) - x_1 f(x_o)}{(fx_1) - f(x_o)}$

$x_2 = \frac{(1)(-0.375) - (0.5)(-1)}{(-0.375) - (-1)}$

$x_2 = \frac{(1)(-0.375) + (0.5)(1)}{(-0.375) + (1)}$

$x_2 = \frac{(-0.375) + (0.5)}{0.625}$

$x_2 = \frac{0.125}{0.625}$

$x_2 = 0.2$

ITERATION 2

n $= 2$

$f(x_2) = f(0.2) = 0.408$

$x_3 = \dfrac{x_1(fx_2) - x_2 f(x_1)}{(fx_2) - f(x_1)}$

$x_3 = \dfrac{(0.5)(0.408) - (0.2)(-0.375)}{(0.408) - (-0.375)}$

$x_3 = \dfrac{(0.204) + (0.075)}{(0.408) + (0.375)}$

$x_3 = \dfrac{0.279}{0.783}$

$x_3 = 0.3563$

ITERATION 3

n $= 3$

$f(x_3) = f(0.3563) = -0.02367$

$x_4 = \dfrac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)}$

$x_4 = \dfrac{(0.2)f(0.3563) - (0.3563)f(0.408)}{f(0.3563) - f(0.408)}$

$x_4 = \dfrac{(0.2)(-0.02367) - (0.3563)(0.408)}{(-0.02367) - (0.408)}$

$x_4 = \dfrac{-0.004737 - 0.1453}{-0.43167}$

$x_4 = \dfrac{-0.150034}{-0.43167}$

$x_4 = 0.3477$

ITERATION 4

$n = 4$

$f(x_3) = f(0.3477) = -0.02367$

$x_4 = 0.3477$ $f(x_4) = -0.00107$

$x_5 = \dfrac{x_3 f(x_4) - x_4 f(x_3)}{f(x_4) - f(x_3)}$

$x_5 = \dfrac{(0.3563)(-0.00107) - (0.3563)(-0.02367)}{(-0.00107) - (-0.02367)}$

$x_5 = \dfrac{-0.000381 + 0.0823}{-0.00107 + 0.02367}$

$x_5 = \dfrac{0.7849}{0.0226}$

$x_5 = 0.3473$

$x_5 = 0.3473$ $f(x_5) = -0.0000096$

Hence, the root after four iterations is   **0.347**

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Question 2

Use the secant method to solve the equation $e^x - 3x^2$ for $0 \le x \le 1$. (Perform only 3 iterations.)

Solution

$f(x) = e^x - 3x^2$, $x_0 = 0$, $x_1 = 1$

both are the initial approximations

$f(0) = e^0 - 3(0) = 1$

$f(1) = e^1 - 3(1) = -0.281$

now we calculate the sec*ond approximation*

$x_2 = \dfrac{x_0 f(x_1) - x_1 f(x_0)}{f(x_0) - f(x_1)} = \dfrac{(0)(-0.281) - (1)1}{-0.281 - 1} = 0.7806$

$f(0.7806) = e^{0.7806} - 3(0.7806)^2 = 2.1827 - 1.82800 = 0.3546$

now $x_1 = 1$ *and* $x_2 = 0.786$

$f(1) = -0.281$ $f(0.786) = 0.3546$

$x_3 = \dfrac{x_1 f(x_2) - x_2 f(x_1)}{f(x_2) - f(x_1)} = \dfrac{1(0.3546) - (0.786)(-0.281)}{0.3546 + 0.281} = 0.9052$

now

$x_3 = 0.9052$ $f(0.9052) = 2.71^{0.9052} - 3(0.9052)^2 = 0.0074$

$x_2 = 0.786$ $f(0.786) = 0.3546$

$x_4 = \dfrac{x_2 f(x_3) - x_3 f(x_2)}{f(x_3) - f(x_2)} = \dfrac{0.7806(0.0074) - (0.9052)(0.3546)}{0.0074 + 0.3546} = 0.9076$

so root is 0.9076

Question 1

Solve the equation $x^3 - 7x^2 + 14x - 6$ by using Muller's method only perform three Iterations.(

$x_0 = 0.5, x_1 = 1, x_2 = 0$)

Solution

1st iteration

$x_0 = 0.5$, $x_1 = 1$ and $x_2 = 0$

$$f(x_0) = f_0 = f(0.5) = (0.5)^3 - 7(0.5)^2 + 14(0.5) - 6 = -0.625$$
$$f(x_1) = f_1 = f(1) = 1^3 - 7(1)^2 + 14(1) - 6 = 2$$
$$f(x_2) = f_2 = f(0) = (0)^3 - 7(0)^2 + 14(0) - 6 = -6$$

$$c = f_0 = -0.625$$
$$h_1 = x_1 - x_0 = 1 - 0.5 = 0.5$$
$$h_2 = x_0 - x_2 = 0.5 - 0 = 0.5$$

$$a = \frac{h_2 f_1 - (h_1 + h_2) f_0 + h_1 f_2}{h_1 h_2 (h_1 + h_2)}$$
$$a = \frac{(0.5)(2) - (0.5 + 0.5)(-0.625) + (0.5)(-6)}{(0.5)(0.5)(0.5 + 0.5)} = \frac{1 - (1)(-0.625) + (0.5)(-6)}{(0.5)(0.5)(1)} = -5.5$$

$$b = \frac{f_1 - f_0 - ah_1^2}{h_1}$$
$$b = \frac{2 - (-0.625) - (-5.5)(0.5)^2}{0.5} = \frac{2 + 0.625 + (5.5)(0.25)^2}{0.5} = 8$$

$$x = x_0 - \frac{2c}{b - \sqrt{b^2 - 4ac}}$$
$$x = 0.5 - \frac{2(-0.625)}{8 - \sqrt{(8)^2 - (4)(-5.5)(-0.625)}}$$
$$x = 1.8721$$

2nd iteration

$x_0 = 0.5$, $x_1 = 1.8721$ and $x_2 = 1$

$$f(x_0) = f_0 = f(0.5) = (0.5)^3 - 7(0.5)^2 + 14(0.5) - 6 = -0.625$$
$$f(x_1) = f_1 = f(1) = 1.8721^3 - 7(1.8721)^2 + 14(1.8721) - 6 = 2.228$$

$$f(x_2) = f_2 = f(0) = (0)^3 - 7(0)^2 + 14(0) - 6 = -6$$

$$c = f_0 = -0.625$$
$$h_1 = x_1 - x_0 = 1.8721 - 0.5 = 1.3721$$
$$h_2 = x_0 - x_2 = 0.5 - 1 = -0.5$$

$$a = \frac{h_2 f_1 - (h_1 + h_2) f_0 + h_1 f_2}{h_1 h_2 (h_1 + h_2)}$$

$$a = \frac{(-0.5)(2.228) - (1.3721 - 0.5)(-0.625) + (1.3721)(-6)}{(1.3721)(-0.5)(1.3721 - 0.50.5)} = 14.70$$

$$b = \frac{f_1 - f_0 - a h_1^2}{h_1}$$

$$b = \frac{2.228 - (-0.625) - (14.70)(1.3721)^2}{0.5} = -17.72$$

$$x = x_0 - \frac{2c}{b - \sqrt{b^2 - 4ac}}$$

$$x = 0.5 + \frac{1.25}{12.62 - \sqrt{(17.72)^2 - (4)(14.71)(-0.625)}} = 0.29$$

$$x = 1.8721$$

For 3rd **iteration**
$$x_0 = 0.29, \ x_1 = 1.8721, \ x_2 = 1$$

For third iteration we will proceed in the same manner.

Graeffe's Root Square Method

Linear Equations

Gaussian Elimination Method

Question 2
Using Gaussian Elimination Method, solve the following system of equations
$$x_1 + x_2 + 2x_3 = 9$$
$$x_1 + 3x_2 + 2x_3 = 13$$
$$3x_1 + x_2 + 3x_3 = 14$$

Solution:

Marks: 10

The Augmented Matrix of the given system of equations is

$$A_b = \begin{bmatrix} 1 & 1 & 2 & 9 \\ 1 & 3 & 2 & 13 \\ 3 & 1 & 3 & 14 \end{bmatrix}$$

$$\square^R \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 2 & 0 & 4 \\ 0 & -2 & -3 & -13 \end{bmatrix} \quad by \quad R_2 - R_1 \ and \ R_3 - 3R_1$$

$$\square^R \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & -2 & -3 & -13 \end{bmatrix} \quad by \quad \frac{1}{2}R_2$$

$$\square^R \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & -3 & -9 \end{bmatrix} \quad by \quad R_3 + (-2)R_2$$

$$\square^R \begin{bmatrix} 1 & 1 & 2 & 9 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix} \quad by \quad \frac{-1}{3}R_3$$

Which shows that from the third and second rows

Z=3, y = 2
And from the first row

X+y+2z=9

Using the values of y and z, we get   x = 1
Hence the solution of the given system is
X = 1, y = 2, z = 3
Question 1
Using Gaussian Elimination Method, solve the following system of equations
$$2x - y + 2z = 2$$
$$x + 10y - 3z = 5$$
$$x - y - z = 3$$

Marks: 10

Solution:
The Augmented Matrix of the given system of equations is

$$A_b = \begin{bmatrix} 2 & -1 & 2 & 2 \\ 1 & 10 & -3 & 5 \\ 1 & -1 & -1 & 3 \end{bmatrix}$$

$$\square^R \begin{bmatrix} 1 & 10 & -3 & 5 \\ 2 & -1 & 2 & 2 \\ 1 & -1 & -1 & 3 \end{bmatrix} \quad by \quad R_{12}$$

$$\square^R \begin{bmatrix} 1 & 10 & -3 & 5 \\ 0 & -21 & 8 & -8 \\ 0 & -11 & 2 & -2 \end{bmatrix} \quad by \quad R_2 - 2R_1, R_3 - R_1$$

$$\square^R \begin{bmatrix} 1 & 10 & -3 & 5 \\ 0 & 1 & -\dfrac{8}{21} & \dfrac{8}{21} \\ 0 & -11 & 2 & -2 \end{bmatrix} \quad by \quad \dfrac{-1}{21}R_2$$

$$\square^R \begin{bmatrix} 1 & 10 & -3 & 5 \\ 0 & 1 & -\dfrac{8}{21} & \dfrac{8}{21} \\ 0 & 0 & -\dfrac{46}{21} & \dfrac{46}{21} \end{bmatrix} \quad by \quad R_3 + 11R_2$$

Using Gaussian Elimination method, by backward substitution, we get as follows
From the third row, we get

$$-\frac{46}{21}z = \frac{46}{21}$$

$$\Rightarrow z = -1$$

From the second row, we get

$$y - \frac{8}{21}z = \frac{8}{21}$$

$$\Rightarrow y = \frac{8}{21} + \frac{8}{21}z$$

Putting the value of z, we get

$$y = \frac{8}{21} + \frac{8}{21}(-1)$$

$$= \frac{8}{21} - \frac{8}{21}$$

$$= 0$$

And finally from the first row, we get
$$x + 10y - 3z = 5$$

Putting the values of y and z, we get
$$x + 10(0) - 3(-1) = 5$$

$$\Rightarrow x + 3 = 5$$

$$\Rightarrow x = 2$$

So, the solution is

$$x = 2, \ y = 0, \ z = -1$$

Guass-Jordan Elimination Method

Question#1   **Marks 10**

By using Gauss –Jordan elimination method, solve the following system of equations,

$$x + y + z = 7$$
$$3x + 3y + 4z = 24$$
$$2x + y + 3z = 16$$

Solution:

The given system in matrix form is

$$\begin{bmatrix} 1 & 1 & 1 \\ 3 & 3 & 4 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 7 \\ 24 \\ 16 \end{bmatrix}$$

$$A \quad\quad X \ = \ B$$

$$\therefore [A \,|\, B] = \begin{bmatrix} 1 & 1 & 1 & : & 7 \\ 3 & 3 & 4 & : & 24 \\ 2 & 1 & 3 & : & 16 \end{bmatrix}$$

$$\square \begin{bmatrix} 1 & 1 & 1 & : & 7 \\ 2 & 1 & 3 & : & 16 \\ 3 & 3 & 4 & : & 24 \end{bmatrix} \quad\quad R_{23}$$

$$\square \begin{bmatrix} 1 & 1 & 1 & : & 7 \\ 0 & -1 & 1 & : & 2 \\ 0 & 0 & 1 & : & 3 \end{bmatrix} \quad\quad R_2 \rightarrow R_2 - 2R_1, R_3 \rightarrow R_3 - 3R_1$$

$$\square \begin{bmatrix} 1 & 0 & 2 & : & 9 \\ 0 & -1 & 1 & : & 2 \\ 0 & 0 & 1 & : & 3 \end{bmatrix} \quad\quad R_1 \rightarrow R_1 + R_2$$

$$\square \begin{bmatrix} 1 & 0 & 0 & : & 3 \\ 0 & -1 & 0 & : & -1 \\ 0 & 0 & 1 & : & 3 \end{bmatrix} \quad\quad R_1 \rightarrow R_1 - 2R_3, R_2 \rightarrow R_2 - R_3$$

$$\square \begin{bmatrix} 1 & 0 & 0 & : & 3 \\ 0 & 1 & 0 & : & 1 \\ 0 & 0 & 1 & : & 3 \end{bmatrix} \quad\quad -R_2$$

So,
$$x = 3, y = 1, z = 3$$

Jacobi's Iterative Method

Solve the following system of equations

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

By Jacobi's iterative method taking the initial starting of solution vector as $(0,0,0)^T$ and perform the first three iterations.

Solution:

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$
$$x = \frac{17 - y + 2z}{20}$$
$$y = \frac{-18 - 3x + z}{20}$$
$$z = \frac{25 - 2x + 3y}{20}$$

starting with $(0,0,0)$

Iteration#01

$$x = \frac{17 - 0 + 2(0)}{20} = \frac{17}{20} = 0.85$$
$$y = \frac{-18 - 3(0) + 0}{20} = \frac{-18}{20} = -0.9$$
$$z = \frac{25 - 2(0) + 3(0)}{20} = \frac{25}{20} = 1.25$$

Iteration#02
$x = 0.85, y = -0.9, z = 1.25$

$$x = \frac{17 - (-0.9) + 2(1.25)}{20} = \frac{20.4}{20} = 1.02$$
$$y = \frac{-18 - 3(0.85) + 1.25}{20} = \frac{-19.3}{20} = -0.965$$
$$z = \frac{25 - 2(0.85) + 3(-0.9)}{20} = \frac{20.6}{20} = 1.03$$

Iteration#03
$x = 1.02, y = -0.965, z = 1.03$

$$x = \frac{17 - (-0.965) + 2(1.03)}{20} = \frac{20.025}{20} = 1.00125$$

$$y = \frac{-18 - 3(1.02) + 1.03}{20} = \frac{-20.03}{20} = -1.0015$$

$$z = \frac{25 - 2(1.02) + 3(-0.965)}{20} = \frac{20.065}{20} = 1.00325$$

Gauss-Seidel Iteration Method

Question#3   **Marks 10**

Solve Question No. #2 by Gauss-Seidel iterative method and perform first three iterations. What you see the difference after solving the same question by two different iterative methods? Give your comments.

Solution:

The above system of linear equations is diagonally dominant; therefore, Gauss-Seidel iterative method could be applied to find out real roots

$$20x + y - 2z = 17$$
$$3x + 20y - z = -18$$
$$2x - 3y + 20z = 25$$

The above system of equations could be written in the form

$$x = \frac{17 - y + 2z}{20}$$
$$y = \frac{-18 - 3x + z}{20}$$
$$z = \frac{25 - 2x + 3y}{20}$$

starting with $(0,0,0)$

Iteration#01

$$x = \frac{17 - 0 + 2(0)}{20} = \frac{17}{20} = 0.85$$

$$y = \frac{-18 - 3(0.85) + 0}{20} = \frac{-20.55}{20} = -1.0275$$

$$z = \frac{25 - 2(0.85) + 3(-1.0275)}{20} = \frac{20.2175}{20} = 1.010875$$

Iteration#02

$$x = 0.85, \; y = -1.0275, \; z = 1.010875$$

$$x = \frac{17 - (-1.0275) + 2(1.010875)}{20} = \frac{20.04925}{20} = 1.0024625$$

$$y = \frac{-18 - 3(1.0024625) + 1.010875}{20} = \frac{-19.9965125}{20} = -0.999825625$$

$$z = \frac{25 - 2(1.0024625) + 3(-0.999825625)}{20} = \frac{19.995598125}{20} = 0.99977990625$$

Iteration#03

$$x = 1.0024625, \, y = -0.999825625, \, z = 0.99977990625$$

$$x = \frac{17 - (-0.999825625) + 2(0.99977990625)}{20} = \frac{19.9904160625}{20} = 0.9999708$$

$$y = \frac{-18 - 3(0.9999708) + 0.99977990625}{20} = \frac{-20.00013249375}{20} = -1.0000066247$$

$$z = \frac{25 - 2(0.9999708) + 3(-1.0000066247)}{20} = \frac{20.000038526}{20} = 1.0000019263$$

In Gauss Siedal Method, the newly computed values in each iteration are directly involved to find the other value of the system of equation and save memory for computation and hence results are more accurate.

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Question 2

Do five iterations to solve the following system of equations by Gauss-Seidal iterative method

$$10x - 2y - 3z = 305$$
$$-2x + 10y - 2z = 154$$
$$-2x - y + 10z = 120$$

Marks: 10

Solution:

Since, the given system is diagonally dominant; hence we can apply here the Gauss-Seidal method.

From the given system of equations

$$x^{r+1} = \frac{1}{10}\left[305 + 2y^r + 3z^r\right]$$

$$y^{r+1} = \frac{1}{10}\left[154 + 2x^{r+1} + 2z^r\right]$$

$$z^{r+1} = \frac{1}{10}\left[120 + 2x^{r+1} + y^{r+1}\right]$$

ITERATION 1

For   r =0

Taking y=z=0 on right hand side of first equation. In second equation we take z=0 and current value of x. In third equation we take current value of both x and y.

$$x^1 = \frac{1}{10}\left[305 + 2(0) + 3(0)\right] = \frac{305}{10} = 30.5$$

$$y^1 = \frac{1}{10}\left[154 + 2(30.5) + 2(0)\right] = \frac{1}{10}\left[154 + 61\right] = \frac{215}{10} = 21.5$$

$$z^1 = \frac{1}{10}\left[120 + 2(30.5) + (21.5)\right] = \frac{1}{10}\left[120 + 61 + 21.5\right] = \frac{202.5}{10} = 20.25$$

ITERATION 2

Similar procedure as used in Iteration 1 will be used for Iterations 2, 3, 4 and 5.

$$x^2 = \frac{1}{10}\left[305 + 2(21.5) + 3(20.25)\right] = \frac{408.75}{10} = 40.875$$

$$y^2 = \frac{1}{10}\left[154 + 2(40.875) + 2(20.25)\right] = \frac{276.25}{10} = 27.625$$

$$z^2 = \frac{1}{10}\left[120 + 2(40.875) + (27.625)\right] = \frac{229.375}{10} = 22.938$$

ITERATION 3

$$x^3 = \frac{1}{10}\left[305 + 2(27.625) + 3(22.938)\right] = \frac{429.064}{10} = 42.906$$

$$y^3 = \frac{1}{10}\left[154 + 2(42.906) + 2(22.938)\right] = \frac{285.688}{10} = 28.569$$

$$z^3 = \frac{1}{10}\left[120 + 2(42.906) + (28.569)\right] = \frac{234.381}{10} = 23.438$$

ITERATION 4

$$x^4 = \frac{1}{10}\left[305 + 2(28.569) + 3(23.438)\right] = \frac{432.452}{10} = 43.245$$

$$y^4 = \frac{1}{10}\left[154 + 2(43.245) + 2(23.438)\right] = \frac{287.366}{10} = 28.737$$

$$z^4 = \frac{1}{10}\left[120 + 2(43.245) + (28.737)\right] = \frac{235.227}{10} = 23.523$$

ITERATION 5

$$x^5 = \frac{1}{10}\left[305 + 2(28.737) + 3(23.523)\right] = \frac{433.043}{10} = 43.304$$

$$y^5 = \frac{1}{10}\left[154 + 2(43.304) + 2(23.523)\right] = \frac{287.654}{10} = 28.765$$

$$z^5 = \frac{1}{10}\left[120 + 2(43.304) + (28.765)\right] = \frac{235.373}{10} = 23.537$$

Above Results are summarized in tabular form as

Iterations	Variables		
	x	y	z
1	30.5	21.5	20.25
2	40.875	27.625	22.938
3	42.906	28.569	23.438

| 4 | 43.245 | 28.737 | 23.523 |
| 5 | 43.304 | 28.765 | 23.537 |

Hence the solution of the given system of equations, after five iterations, is

$x = 43.304$

$y = 28.765$

$z = 23.537$

Relaxation Method

Matrix Inversion

Eigen Value Problems

Power Method

Question 1

Find the largest eigen value of the matrix

$$\begin{pmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 0 & 4 \end{pmatrix}$$

MARKS   10

And the corresponding eigenvector, by Power Method after fourth iteration starting with the initial vector $v^{(0)} = (0,0,1)^T$

SOLUTION

Let $A = \begin{pmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 0 & 4 \end{pmatrix}$

Choosing an initial vector as

$$v^{(0)} = (0,0,1)^T \quad then$$

 ITERATION 1

$$u^{(1)} = [A]v^{(0)} \begin{pmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \\ 4 \end{pmatrix}$$

Now we normalize the resultant vector to get

$$u^{(1)} = 4 \begin{pmatrix} 0 \\ \dfrac{1}{4} \\ 1 \end{pmatrix} = q_1 v^{(1)}$$

Continuing this procedure for subsequent Iterations , we have

 ITERATION 2

$$u^{(2)} = [A]v^{(1)} \begin{pmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} 0 \\ \dfrac{1}{4} \\ 1 \end{pmatrix} = \begin{pmatrix} \dfrac{1}{4} \\ 6 \\ 4 \end{pmatrix}$$

$$u^{(2)} = 6 \begin{pmatrix} \dfrac{1}{24} \\ 1 \\ \dfrac{2}{3} \end{pmatrix} = q_2 v^{(2)}$$

ITERATION 3

$$u^{(3)} = [A]v^{(2)} \begin{pmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} \dfrac{1}{24} \\ 1 \\ \dfrac{2}{3} \end{pmatrix} = \begin{pmatrix} 1.167 \\ 20.708 \\ 2.667 \end{pmatrix}$$

$$u^{(3)} = 20.708 \begin{pmatrix} 0.056 \\ 1 \\ 0.129 \end{pmatrix} = q_3 v^{(3)}$$

ITERATION 4

$$u^{(4)} = [A]v^{(3)} \begin{pmatrix} 4 & 1 & 0 \\ 1 & 20 & 1 \\ 0 & 0 & 4 \end{pmatrix} \begin{pmatrix} 0.056 \\ 1 \\ 0.129 \end{pmatrix} = \begin{pmatrix} 1.224 \\ 20.185 \\ 0.516 \end{pmatrix}$$

$$u^{(4)} = 20.185 \begin{pmatrix} 0.061 \\ 1 \\ 0.026 \end{pmatrix} = q_4 v^{(4)}$$

Therefore, the largest eigen value and the corresponding eigen vector accurate to three decimals places are

$\lambda = 20.185$

$$(X) = \begin{pmatrix} 0.061 \\ 1 \\ 0.026 \end{pmatrix}$$

===

Question 1   **marks 10**

Find the largest eigen value of the matrix
$$\begin{bmatrix} 6 & 5 & 2 \\ 3 & 1 & 4 \\ 1 & 10 & 3 \end{bmatrix}$$

And the corresponding eigen vector, by Power Method after fourth iteration starting with the initial vector $v^{(0)} = (0,1,0)^T$

SOLUTION

Let $A = \begin{pmatrix} 6 & 5 & 2 \\ 3 & 1 & 4 \\ 1 & 10 & 3 \end{pmatrix}$

Choosing an initial vector
$$v^{(0)} = (0,1,0)^T \text{ then}$$

ITERATION 1

$$u^{(1)} = [A]v^{(0)} = \begin{pmatrix} 6 & 5 & 2 \\ 3 & 1 & 4 \\ 1 & 10 & 3 \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 5 \\ 1 \\ 10 \end{pmatrix}$$

Now we normalize the resultant vector to get
$$u^{(1)} = 10 \begin{pmatrix} 0.5 \\ 0.1 \\ 1 \end{pmatrix} = q_1 v^{(1)}$$

Continuing this procedure for subsequent Iterations, we have
ITERATION 2

$$u^{(2)} = [A]v^{(1)} = \begin{pmatrix} 6 & 5 & 2 \\ 3 & 1 & 4 \\ 1 & 10 & 3 \end{pmatrix} \begin{pmatrix} 0.5 \\ 0.1 \\ 1 \end{pmatrix} = \begin{pmatrix} 5.5 \\ 5.6 \\ 4.5 \end{pmatrix}$$

$$u^{(2)} = 5.6 \begin{pmatrix} \dfrac{5.5}{5.6} \\ \dfrac{5.6}{5.6} \\ \dfrac{4.5}{5.6} \end{pmatrix} = \begin{pmatrix} 0.982 \\ 1 \\ 0.804 \end{pmatrix} = q_2 v^{(2)}$$

ITERATION 3

$$u^{(3)} = [A]v^{(2)} = \begin{pmatrix} 6 & 5 & 2 \\ 3 & 1 & 4 \\ 1 & 10 & 3 \end{pmatrix} \begin{pmatrix} 0.982 \\ 1 \\ 0.804 \end{pmatrix} = \begin{pmatrix} 12.5 \\ 7.162 \\ 13.394 \end{pmatrix}$$

$$u^{(3)} = 13.394 \begin{pmatrix} 0.933 \\ 0.535 \\ 1 \end{pmatrix} = q_3 v^{(3)}$$

ITERATION 4

$$u^{(4)} = [A]v^{(3)} = \begin{pmatrix} 6 & 5 & 2 \\ 3 & 1 & 4 \\ 1 & 10 & 3 \end{pmatrix} \begin{pmatrix} 0.933 \\ 0.535 \\ 1 \end{pmatrix} = \begin{pmatrix} 10.273 \\ 7.334 \\ 9.283 \end{pmatrix}$$

$$u^{(4)} = 10.273 \begin{pmatrix} 1 \\ 0.714 \\ 0.904 \end{pmatrix} = q_4 v^{(4)}$$

Therefore, the largest eigen value and the corresponding eigen vector accurate to four decimals places are

$\lambda = 10.273$

and

$$(X) = \begin{pmatrix} 1 \\ 0.714 \\ 0.904 \end{pmatrix}$$

Jacobi's Method

Question 1

Using Jacobi's method, find all the eigenvalues and the corresponding eigenvectors of the following matrix,

$$\begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

Marks: 10

Note: Give results at the end of third rotation.

Solution.

$$Let \ A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

$$Then, \ A^t = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$$

Hence,

Matrix A is real and Symmetric and Jacobi's method can be applied.

Rotation 1

In Matrix A, all the off-diagonal elements are found be 2.
So, the largest off-diagonal element is found to be
$a_{12} = a_{13} = a_{21} = a_{23} = a_{31} = a_{32} = 2$.

Therefore, we can choose any one of them as the largest element.

Suppose, we choose a_{12} as the largest element

Then, we compute the rotation angle as,

$$\tan 2\theta = \frac{2a_{12}}{a_{11} - a_{22}} = \frac{2 \times 2}{1 - 1} = \infty$$

Therefore

$\tan 2\theta = \infty$

$$\Rightarrow \quad 2\theta = \frac{\pi}{2}$$

$$\Rightarrow \quad \boldsymbol{\theta = \frac{\pi}{4}}$$

Therefore, we construct an orthogonal matrix S_1 such that,

$$S_1 = \begin{bmatrix} \cos\frac{\pi}{4} & -\sin\frac{\pi}{4} & 0 \\ \sin\frac{\pi}{4} & \cos\frac{\pi}{4} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now the first rotation gives,

$$D_1 = S_1^{-1} A S_1$$

Here, $S_1^{-1} = = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$

So,

$D_1 = S_1^{-1} A S_1$

i.e.

$$D_1 = \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 0 & 2.828 \\ 0 & -1 & 0 \\ 2.828 & 0 & 1 \end{bmatrix}$$

To check that we are right in our calculations, we can see that the sum of the diagonal elements is, $3 + (-1) + (1) = 3$, which is same as the sum of the diagonal elements of the original matrix A.

Rotation 2

For the second rotation we choose the largest off-diagonal element $d_{13} = d_{31} = 2.828$, then

$$\tan 2\theta = \frac{2d_{13}}{d_{11} - d_{33}} = \frac{2(2.828)}{3 - (1)} = \frac{5.657}{2} = 2.828$$

So,

$2\theta = \tan^{-1}(2.828) = 70.526$

$\theta = 35.263$

Therefore, we construct an orthogonal matrix S_2 such that,

$$S_2 = \begin{bmatrix} \cos(35.263) & 0 & -\sin(35.263) \\ 0 & 1 & 0 \\ \sin(35.263) & 0 & \cos(35.263) \end{bmatrix}$$

$$= \begin{bmatrix} 0.817 & 0 & -0.577 \\ 0 & 1 & 0 \\ 0.577 & 0 & 0.817 \end{bmatrix}$$

$$S_2^{-1} = \; = \begin{bmatrix} 0.817 & 0 & 0.577 \\ 0 & 1 & 0 \\ -0.577 & 0 & 0.817 \end{bmatrix}$$

Now the rotation 2 gives,

$$D_2 = S_2^{-1} D_1 S_2$$

$$D_2 = \begin{bmatrix} 0.817 & 0 & 0.577 \\ 0 & 1 & 0 \\ -0.577 & 0 & 0.817 \end{bmatrix} \begin{bmatrix} 3 & 0 & 2.828 \\ 0 & -1 & 0 \\ 2.828 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.817 & 0 & -0.577 \\ 0 & 1 & 0 \\ 0.577 & 0 & 0.817 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$$

Again to check that we are right in our calculations, we can see that the sum of the diagonal elements is $5 + (-1) + (-1) = 3$ which is same as the sum of the diagonal elements of the original matrix A.

Rotation 3
We can see that in above iteration that D_2 is a diagonal matrix, so we stop here and take the **Eigen Values** as
$\lambda_1 = 5$, $\lambda_2 = -1$, $\lambda_3 = -1$

Now the Eigenvectors are the columns vectors of the matrix $S = S_1 S_2$, which are,

$$S = \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0.817 & 0 & -0.577 \\ 0 & 1 & 0 \\ 0.577 & 0 & 0.817 \end{bmatrix}$$

$$= \begin{bmatrix} 0.577 & -0.707 & -0.408 \\ 0.577 & 0.707 & -0.408 \\ 0.577 & 0 & 0.817 \end{bmatrix}$$

Therefore, the corresponding **EigenVectors** are

$$X_1 = \begin{bmatrix} 0.577 \\ 0.577 \\ 0.577 \end{bmatrix}, \qquad X_2 = \begin{bmatrix} -0.707 \\ 0.707 \\ 0 \end{bmatrix}, \qquad X_3 = \begin{bmatrix} -0.408 \\ -0.408 \\ 0.817 \end{bmatrix}$$

Interpolation

For a given table of values (,), 0,1,2,..., k k x y k= nwith equally spaced abscissas of a function y= f(x),we define the forward difference operator Δ as follows,

$$\Delta y_i = y_{i+1} - y_i, \qquad i = 0,1,...,(n-1)$$

These differences are called first differences of the function y and are denoted by the symbol Δy_i Here, Δ is called the first difference operator.

Similarly, rth Difference operator would be $\Delta^r y_i = \Delta^{r-1} y_{i+1} - \Delta^{r-1} y_i$

Leading term =y_o

Leading Diffenrence=Δy

Backward Difference Operators:

$$\nabla y_i = y_i - y_{i-1} \quad i = n, \ (n-1),...,1$$

Central Difference is given by,

$$\delta y_i = y_{i+(1/2)} - y_{i-(1/2)}$$

$$\Delta y_0 = y_1 - y_0$$
$$\Delta y_1 = y_2 - y_1$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$\Delta y_{n-1} = y_n - y_{n-1}$$

$$\nabla y_i = y_i - y_{i-1} \; i = n, \;\; (n-1), \ldots, 1$$

OR

$$\nabla y_1 = y_1 - y_0$$
$$\nabla y_2 = y_2 - y_1$$
$$\vdots \qquad \vdots \qquad \vdots$$
$$\nabla y_n = y_n - y_{n-1}$$

$$\delta y_{1/2} = y_1 - y_0, \qquad \delta y_{3/2} = y_2 - y_1,$$

In General,

$$\delta y_i = y_{i+(1/2)} - y_{i-(1/2)}$$

Higher order differences are defined as follows:

$$\delta^2 y_i = \delta y_{i+(1/2)} - \delta y_{i-(1/2)}$$

$$\delta^n y_i = \delta^{n-1} y_{i+(1/2)} - \delta^{n-1} y_{i-(1/2)}$$

Shift operator, E

Let y = f (x) be a function of x, and let x takes the consecutive values x, x + h, x + 2h, etc. We then define an operator having the property

E f(x) = f(x+h)

Thus, when E operates on f (x), the result is the next value of the function. Here, E is called the shift operator. If we apply the operator E twice on f (x), we get

$$E^2 f(x) = E[E f(x)]$$

$$= E[f(x+h)] = f(x+2h)$$

Thus, in general, if we apply the operator 'E' n times on f (x), we get

E^nf(x)= f (x+nh)

The inverse operator E is defined as

$$E^{-1} f(x) = f(x-h)$$

Similarly

$$E^{-n} f(x) = f(x-nh)$$

Average Operator, μ;

it is defined as

$$\mu f(x) = \frac{1}{2} \left[f\left(x + \frac{h}{2} \right) + f\left(x - \frac{h}{2} \right) \right]$$

$$= \frac{1}{2} \left[y_{x+(h/2)} + y_{x-(h/2)} \right]$$

Differential Operator, D

it is defined as

$$Df(x) = \frac{d}{dx} f(x) = f'(x)$$

$$D^2 f(x) = \frac{d^2}{dx^2} f(x) = f''(x)$$

$$\Rightarrow \Delta = E - 1$$

$$\Rightarrow \nabla = 1 - E^{-1} = \frac{E-1}{E}$$

$$\delta = E^{1/2} - E^{-1/2}$$

$$\Rightarrow \mu = \frac{1}{2}(E^{1/2} + E^{-1/2})$$

$$hD = \log E$$

Newton Forward Difference Interpolation.

for any real number p, we have the operator E such that

$$E^p f(x) = f(x + ph).$$

$$f(x_0 + ph) = E^p f(x_0) = (1+\Delta)^p f(x_0)$$

$$= \left[1 + p\Delta + \frac{p(p-1)}{2!}\Delta^2 + \frac{p(p-1)(p-2)}{3!}\Delta^3 + \cdots \right] f(x_0)$$

$$f(x_0 + ph) = f(x_0) + p\Delta f(x_0)$$

$$+ \frac{p(p-1)}{2!}\Delta^2 f(x_0) + \frac{p(p-1)(p-2)}{3!}\Delta^3 f(x_0)$$

$$+ \cdots + \frac{p(p-1)\cdots(p-n+1)}{n!}\Delta^n f(x_0) + \text{Error}$$

Forward Differences

Question 2

Construct a forward difference table from the following values of x and y.

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y=f(x)	2.0	3.5	-1.7	2.3	4.2	6.5	5.7

Marks: 10

Solution.

Forward-difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
1.0	2						
		1.5					

1.2	3.5		-6.7				
		-5.2		15.9			
1.4	-1.7		9.2		-27.2		
		4		-11.3		41	
1.6	2.3		-2.1		13.8		-60.8
		1.9		2.5		-19.8	
1.8	4.2		0.4		-6		
		2.3		-3.5			
2.0	6.5		-3.1				
		-0.8					
2.2	5.7						

Backward Differences

Question 2

Construct a backward difference table from the following values of x and y
.

x	1.0	1.2	1.4	1.6	1.8	2.0	2.2
y=f(x)	2.0	3.5	-1.7	2.3	4.2	6.5	5.7

Marks: 10

Solution.

Backward-difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
1.0	2						
		1.5					
1.2	3.5		-6.7				
		-5.2		15.9			
1.4	-1.7		9.2		-27.2		
		4		-11.3		41	
1.6	2.3		-2.1		13.8		-60.8
		1.9		2.5		-19.8	
1.8	4.2		0.4		-6		
		2.3		-3.5			

2.0	6.5		-3.1				
		-0.8					
2.2	5.7						

==

Question 2	marks 10

Form a table of forward and backward differences of the function

$f(x) = x^3 - 3x^2 - 5x - 7$ For $x = -1, 0, 1, 2, 3, 4, 5$

SOLUTION

For the given function, the values of y for the given values of x are calculated as

for x = -1
$f(-1) = (-1)^3 - 3(-1)^2 - 5(-1) - 7 = -6$
for x = 0
$f(0) = (0)^3 - 3(0)^2 - 5(0) - 7 = -7$
for x = 1
$f(1) = (1)^3 - 3(1)^2 - 5(1) - 7 = -14$
for x = 2
$f(2) = (2)^3 - 3(2)^2 - 5(2) - 7 = -21$
for x = 3
$f(3) = (3)^3 - 3(3)^2 - 5(3) - 7 = -22$
for x = 4
$f(4) = (4)^3 - 3(4)^2 - 5(4) - 7 = -11$
for x = 5
$f(5) = (5)^3 - 3(5)^2 - 5(5) - 7 = 18$

So, the table of values of x and y is

x	-1	0	1	2	3	4	5
$y = f(x)$	-6	-7	-14	-21	-22	-11	18

Forward difference table

Forward difference table for the table of values of x and $y = f(x)$ is shown below

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
-1	-6						
		-1					

0	-7		-6				
		-7		6			
1	-14		0		0		
		-7		6		0	
2	-21		6		0		0
		-1		6		0	
3	-22		12		0		
		11		6			
4	-11		18				
		29					
5	18						

Backward difference table

Backward difference table for the table of values of x and $y = f(x)$ is shown below

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$	$\nabla^6 y$
-1	-6						
		-1					
0	-7		-6				
		-7		6			
1	-14		0		0		
		-7		6		0	
2	-21		6		0		0
		-1		6		0	
3	-22		12		0		
		11		6			
4	-11		18				
		29					
5	18						

Divided Differences

Question 1   marks 10

For the following table of values, estimate $f(2.5)$ using Newton's forward difference interpolation formula.

x	1	2	3	4	5	6	7	8
$y = f(x)$	1	8	27	64	125	216	343	512

SOLUTION

Forward difference table

Forward difference table for the given values of x and y is shown below

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
1	1						
		7					
2	8		12				
		19		6			
3	27		18		0		
		37		6		0	
4	64		24		0		0
		61		6		0	
5	125		30		0		
		91		6			
6	216		36				
		127					
7	343		42				
		169					
8	512						

Newton's forward difference interpolation formula is given by

$$y_x = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$+ \frac{p(p-1)(p-2)(p-3)}{4!}\Delta^4 y_0 + \cdots + \frac{p(p-1)(p-2)........(p-n+1)}{n!}\Delta^n y_0$$

Here
$$p = \frac{x-x_0}{h} = \frac{2.5-1}{1} = \frac{1.5}{1} = 1.5$$
And
$$y_0 = 1, \Delta y_0 = 7, \Delta^2 y_0 = 12, \Delta^3 y_0 = 6, \Delta^4 y_0 = \Delta^5 y_0 = \Delta^6 y_0 = 0$$

So, by putting the above values in Newton's forward difference interpolation formula,
We have

$$y_x = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0$$

$$= 1 + 1.5(7) + \frac{1.5(1.5-1)}{2!}(12) + \frac{1.5(1.5-1)(1.5-2)}{3!}(6)$$

$$= 1 + 10.5 + 4.5 - 0.375$$

$$= 15.625$$

i.e.

$$y_{2.5} = 15.625$$

==

| **Question 2** | **marks 10** |

Compute $f(1.5)$ for the following data by using Newton's divided difference interpolation formula.

x	1	2	4	5	8
$f(x)$	5	14	28	45	92

SOLUTION
The divided difference table for the given data is given by

x	y	1st D.D.	2nd D.D.	3rd D.D.	4th D.D.
1	5				
		9			
2	14		-2/3		
		7		1	
4	28		10/3		-29/126
		17		-11/18	
5	45		-1/3		
		47/3			
8	92				

Newton's Divided Difference formula is

$$f(x) = y_0 + (x-x_0)y[x_0,x_1] + (x-x_0)(x-x_1)y[x_0,x_1,x_2] + (x-x_0)(x-x_1)(x-x_2)y[x_0,x_1,x_2,x_3]$$
$$+ (x-x_0)(x-x_1)(x-x_2)(x-x_3)y[x_0,x_1,x_2,x_3,x_4]$$

$$f(1.5) = 5 + (1.5 - 1)\,9 + (1.5-1)(1.5-2)(-\frac{2}{3}) + (1.5-1)\,(1.5-2)(1.5-4)1$$

$$+(1.5-1)\,(1.5-2)(1.5-4)(1.5-5)(-\frac{29}{126})$$

$$=5+4.5+0.1667+0.625+0.5035$$

$$=10.7952$$

Question 3   **marks 10**

Find $y'(6)\ and\ y''(6)$ from the following table of values.

x	1	2	3	4	5	6
y	3	9	17	27	40	55

SOLUTION

Backward difference table

Backward difference table for the given values of x and y is shown below

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
1	3					
		6				
2	9		2			
		8		0		
3	17		2		1	
		10		1		-3
4	27		3		-2	
		13		-1		
5	40		2			
		15				
6	55					

By backward difference formula for first derivative, we have

$$y'_n = \frac{1}{h}\left(\nabla y_n + \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3} + \frac{\nabla^4 y_n}{4}\right)$$

$$y'_n = \frac{1}{h}\left(\nabla y_n + \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3} + \frac{\nabla^4 y_n}{4} + \frac{\nabla^5 y_n}{5}\right)$$

$$y'(6) = \frac{1}{1}\left(15 + \frac{2}{2} + \frac{-1}{3} + \frac{-2}{4} + \frac{-3}{5}\right)$$

$$= 15 + 1 - 0.333 - 0.5 - 0.6$$

$$= 14.567$$

By backward difference formula for second derivative, we have

$$y''_n = \frac{1}{h^2}\left(\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \frac{5}{6}\nabla^5 y_n\right)$$

Therefore

$$y''(6) = \frac{1}{1^2}[2 + (-1) + \frac{11}{12}(-2) + \frac{5}{6}(-3)]$$

$$= 2 - 1 - 1.833 - 2.5$$

$$= -3.333$$

Question#1   Marks 10

Find an equation of a cubic curve which passes through the points
(4,-43), (7, 83), (9,327) and (12, 1053) using Divided Difference Formula.

Solution:

Newton's divided difference table is

X	Y	1st D.D	2nd D.D	3rd D.D
4	-43			
7	83	42		
9	327	122	16	
12	1053	242	24	1

Newton's Divided Difference formula is

$$y = f(x) = y_0 + (x - x_0)y[x_0, x_1] + (x - x_0)(x - x_1)y[x_0, x_1, x_2] + ...$$
$$+ (x - x_0)(x - x_1)....(x - x_{n-1})y[x_0, x_1, ...x_n]$$

now putting values in formula

$$y = f(x) = -43 + (x-4)(42) + (x-4)(x-7)(16) + (x-4)(x-7)(x-9)(1)$$
$$= -43 + (x-4)\{42 + 16x - 112 + x^2 - 16x + 63\}$$
$$= -43 + (x-4)(x^2 - 7)$$
$$= -43 + x^3 - 7x - 4x^2 + 28$$
$$= x^3 - 4x^2 - 7x - 15$$

Hence it is required polinomial

Question 2   marks 10

Form a table of forward and backward differences of the function

$f(x) = x^3 - 3x^2 - 5x - 7$ For $x = -1, 0, 1, 2, 3, 4, 5$

SOLUTION

For the given function, the values of y for the given values of x are calculated as

for x = -1

$f(-1) = (-1)^3 - 3(-1)^2 - 5(-1) - 7 = -6$

for x = 0

$f(0) = (0)^3 - 3(0)^2 - 5(0) - 7 = -7$

for x = 1

$f(1) = (1)^3 - 3(1)^2 - 5(1) - 7 = -14$

for x = 2

$f(2) = (2)^3 - 3(2)^2 - 5(2) - 7 = -21$

for x = 3

$f(3) = (3)^3 - 3(3)^2 - 5(3) - 7 = -22$

for x = 4

$f(4) = (4)^3 - 3(4)^2 - 5(4) - 7 = -11$

for x = 5

$f(5) = (5)^3 - 3(5)^2 - 5(5) - 7 = 18$

So, the table of values of x and y is

x	-1	0	1	2	3	4	5
$y = f(x)$	-6	-7	-14	-21	-22	-11	18

Forward difference table

Forward difference table for the table of values of x and $y = f(x)$ is shown below

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
-1	-6						
		-1					
0	-7		-6				
		-7		6			
1	-14		0		0		
		-7		6		0	
2	-21		6		0		0
		-1		6		0	
3	-22		12		0		
		11		6			
4	-11		18				
		29					
5	18						

Backward difference table

Backward difference table for the table of values of x and $y = f(x)$ is shown below

x	y	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$	$\nabla^6 y$
-1	-6						
		-1					
0	-7		-6				
		-7		6			
1	-14		0		0		
		-7		6		0	
2	-21		6		0		0
		-1		6		0	
3	-22		12		0		
		11					

				6			
4	-11		18				
		29					
5	18						

Question 1   Marks 10

Find the interpolating polynomial for the following data by

      a) Newton's Forward Difference Formula

      b)Lagrange's Formula

Hence show that both the methods give raise to the same polynomial.

x	0	1	2	3
y	1	3	7	13

Solution:

By Newton's Forward Difference Formula

The Newton's Forward difference table is given as

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0	1			
		2		
1	3		2	
		4		0
2	7		2	
		6		
3	13			

We have Newton's forward difference interpolation formula as

$$y = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{3!}\Delta^3 y_0 + \ldots \qquad (1)$$

Since, the third and higher order differences are zero; Therefore, Newton's forward difference Interpolation formula reduces to

$$y = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_0 \qquad (2)$$

Here,

$x_0 = 0, \ y_0 = 1, \ \Delta y_0 = 2, \ \Delta^2 y_0 = 2, \ h = 1$

And p is given by

$$p = \frac{x - x_0}{h} = \frac{x - 0}{1} = x$$

Substituting these values in Eq.(2), we have

$$y = 1 + x(2) + \frac{x(x-1)}{2}(2)$$

$$= 1 + 2x + x(x-1)$$

$$= 1 + 2x + x^2 - x$$

$$= x^2 + x + 1$$

Hence, the required polynomial is

$$y = x^2 + x + 1.$$

b) By Lagrange's Formula

Lagrange's Interpolation formula is given by

$$y = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} y_0 + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} y_1$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} y_2 + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} y_3$$

By putting the values in the formula, we get

$$y = \frac{(x-1)(x-2)(x-3)}{(0-1)(0-2)(0-3)}(1) + \frac{(x-0)(x-2)(x-3)}{(1-0)(1-2)(1-3)}(3)$$

$$+ \frac{(x-0)(x-1)(x-3)}{(2-0)(2-1)(2-3)}(7) + \frac{(x-0)(x-1)(x-2)}{(3-0)(3-1)(3-2)}(13)$$

$$y = \frac{x^3 - 6x^2 + 11x - 6}{(-1)(-2)(-3)} + \frac{3x^3 - 15x^2 + 18x}{(1)(-1)(-2)} \qquad y = \frac{x^3 - 6x^2 + 11x - 6}{-6} + \frac{3x^3 - 15x^2 + 18x}{2}$$

$$+ \frac{7(x^3 - 4x^2 + 3x)}{(2)(1)(-1)} + \frac{13(x^3 - 3x^2 + 2x)}{(3)(2)(1)} \qquad + \frac{7x^3 - 28x^2 + 21x}{-2} + \frac{13x^3 - 39x^2 + 26x}{6}$$

$$y = \frac{x^3 - 6x^2 + 11x - 6}{-6} + \frac{3x^3 - 15x^2 + 18x}{2} + \frac{7x^3 - 28x^2 + 21x}{-2} + \frac{13x^3 - 39x^2 + 26x}{6}$$

$$= \frac{-x^3 + 6x^2 - 11x + 6 + 9x^3 - 45x^2 + 54x - 21x^3 + 84x^2 - 63x + 13x^3 - 39x^2 + 26x}{6}$$

$$y = \frac{-x^3 + 9x^3 - 21x^3 + 13x^3 + 6x^2 - 45x^2 + 84x^2 - 39x^2 - 11x + 54x - 63x + 26x + 6}{6}$$

$$= \frac{6x^2 + 6x + 6}{6}$$

$$= \frac{6(x^2 + x + 1)}{6}$$

$$= x^2 + x + 1$$

Thus we have the equation

$$y = x^2 + x + 1$$

This is the same polynomial as obtained in Newton's Forward difference interpolating formula. Hence it is proved that both the methods give rise to the same polynomial

Langrange's Interpolation

Find interpolation polynomial by Lagrange's Formula, with the help of following table,

x	1	2	3	5
f(x)	0	7	26	124

And hence find value of f (6).
Solution:

Lagrange's int *erpolation formula is*

$$y = f(x) = \frac{(x-x_1)(x-x_2)(x-x_3)}{(x_0-x_1)(x_0-x_2)(x_0-x_3)} f(x_0) + \frac{(x-x_0)(x-x_2)(x-x_3)}{(x_1-x_0)(x_1-x_2)(x_1-x_3)} f(x_1)$$

$$+ \frac{(x-x_0)(x-x_1)(x-x_3)}{(x_2-x_0)(x_2-x_1)(x_2-x_3)} f(x_2) + \frac{(x-x_0)(x-x_1)(x-x_2)}{(x_3-x_0)(x_3-x_1)(x_3-x_2)} f(x_3)$$

$$y = f(x) = \frac{(x-2)(x-3)(x-5)}{(1-2)(1-3)(1-5)}(0) + \frac{(x-1)(x-3)(x-5)}{(2-1)(2-3)(2-5)}(7) + \frac{(x-1)(x-2)(x-5)}{(3-1)(3-2)(3-5)}(26)$$

$$+ \frac{(x-1)(x-2)(x-3)}{(5-1)(5-2)(5-3)}(124)$$

$$= 0 + \frac{7(x^3-9x^2+23x-15)}{3} - \frac{26(x^3-8x^2+17x-10)}{4} + \frac{124(x^3-6x^2+11x-6)}{24}$$

$$= \frac{7(x^3-9x^2+23x-15)}{3} - \frac{13(x^3-8x^2+17x-10)}{2} + \frac{31(x^3-6x^2+11x-6)}{6}$$

$$= \frac{14(x^3-9x^2+23x-15)-39(x^3-8x^2+17x-10)+31(x^3-6x^2+11x-6)}{6}$$

$$= \frac{1}{6}(14x^3-126x^2+322x-210-39x^3+312x^2-663x+390+31x^3-186x^2+341x-186)$$

$$= \frac{1}{6}(6x^3-6)$$

$$y = f(x) = x^3 - 1$$

now we have to find $f(6)$ *so*

$$f(6) = x^3 - 1$$
$$= (6)^3 - 1$$
$$= 216 - 1$$
$$= 215$$

Differentiation Using Difference Operators

Question#3   Marks 10

Find $f'(2.5)$ *and* $f''(2.5)$ from the following table,

x	1.3	1.6	1.9	2.2	2.5
f(x)	2.4	2.9	3.2	4.7	6.4

Solution:

Since x=2.5 occur at the end of the table, it is appropriate to use the backward difference formula for the derivation.

Backward difference table is as

x	$f(x)$	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$
1.3	2.4				
1.6	2.9	0.5			
1.9	3.2	0.3	-0.2		
2.2	4.7	1.5	1.2	1.4	
2.5	6.4	1.7	0.2	-1	-2.4

u sin *g the backward difference formula for* $y'(x)$ *and* $y''(x),$ *we have*

$$y_n' = \frac{1}{h}\left(\nabla y_n + \frac{\nabla^2 y_n}{2} + \frac{\nabla^3 y_n}{3} + \frac{\nabla^4 y_n}{4}\right)$$

now putting value from backward diff .table we have

$$y_{2.5}' = \frac{1}{0.3}\left(1.7 + \frac{0.2}{2} + \frac{(-1)}{3} + \frac{(-2.4)}{4}\right)$$

$$y_{2.5}' = \frac{1}{0.3}\left(1.7 + 0.1 - 0.3333 - 0.6\right)$$

$$y_{2.5}' = \frac{1}{0.3}\left(0.8667\right)$$

$$y_{2.5}' = 2.8890$$

$$y_n'' = \frac{1}{h^2}\left(\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n\right)$$

$$y_{2.5}'' = \frac{1}{(0.3)^2}\left(0.2 - 1 + \frac{11}{12}(-2.4)\right)$$

$$y_{2.5}'' = \frac{1}{(0.09)}\left(0.2 - 1 - 2.2\right)$$

$$y_{2.5}'' = \frac{1}{(0.09)}\left(-3\right)$$

$$y_{2.5}'' = -33.33$$

Numerical Integration

Trapezoidal Rule

Question 1
Marks 10

Evaluate the integral

$$\int_0^1 e^{2x}\,dx$$

Using (I) Trapezoidal rule

(2) Simpson's 1/3 rule

By dividing the interval of integration into eight equal parts. Also calculate the percentage error from its true values in both cases.

Solution:

a) **By Trapezoidal Rule**

Since the no. of intervals are eight, so,

$n = 8$

And $a = 0, b = 1$

So, the width of the interval is given by

$h = \dfrac{b-a}{n}$

$= \dfrac{1-0}{8}$

$= \dfrac{1}{8}$

$= 0.125$

Since the integrand is given by

$y = f(x) = e^{2x}$,

Therefore, the table of values will be

x	0	0.125	0.250	0.375	0.500	0.625	0.750	0.875	1.000
$y = f(x)$	1	1.2840	1.6487	2.1170	2.7183	3.4903	4.4817	5.7546	7.3891

Now applying the Trapezoidal rule, we have

$$\int_0^1 e^{2x}\,dx = \frac{h}{2}\left[y_0 + 2(y_1 + y_2 + y_3 + y_4 + y_5 + y_6 + y_7) + y_8\right]$$

$$= \frac{0.125}{2}\left[1 + 2(1.2840 + 1.6487 + 2.1170 + 2.7183 + 3.4903 + 4.4817 + 5.7546) + 7.3891\right]$$

$$= 0.0625\left[8.3891 + 2(21.4946)\right]$$

$$= 0.0625(8.3891 + 42.9892)$$

$$= 0.0625(51.3783)$$

$$= 3.2111$$

Therefore

$$\int_0^1 e^{2x}\,dx = 3.2111$$

b) By Simpson's Rule

Applying Simpson's rule, we have

$$\int_0^1 e^{2x}\,dx = \frac{h}{3}\left[y_0 + 4(y_1 + y_3 + y_5 + y_7) + 2(y_2 + y_4 + y_6) + y_8\right]$$

$$= \frac{0.125}{3}\left[1 + 4[1.2840 + 2.1170 + 3.4903 + 5.7546] + 2[1.6487 + 2.7183 + 4.4817] + 7.3891\right]$$

$$= \frac{0.125}{3}\left[8.3891 + 4(12.6459) + 2(8.8487)\right]$$

$$= \frac{0.125}{3}(8.3891 + 50.5836 + 17.6974)$$

$$= \frac{0.125}{3}(76.6701)$$

$$= 3.1946$$

Therefore

$$\int_0^1 e^{2x}\,dx = 3.1946$$

True value

Bu integration, we have

$$\int_0^1 e^{2x}\,dx = \left[\frac{e^{2x}}{2}\right]_0^1$$

$$= \frac{1}{2}\left[e^{2x}\right]_0^1$$

$$= \frac{1}{2}\left[e^2 - e^0\right]$$

$$= \frac{1}{2}[7.389056099 - 1]$$

$$= 3.1945$$

Therefore the true value is

$$\int_0^1 e^{2x}\,dx = 3.1945$$

Percentage error in Trapezoidal rule

Error = $\left|Exact\,value - Computed\,value\right|$

So,

$$Error = |3.1945 - 3.2111|$$

$$= |-0.0166|$$

$$= 0.0166$$

And % error is given by

$$\% \ error = \frac{error}{exact \ value} \times 100\%$$

$$= \frac{0.0166}{3.1945} \times 100\%$$

$$= 0.52\%$$

Percentage error in Simpson's 1/3 rule

$$Error = \left| Exact \ value - Computed \ value \right|$$

$$Error = \left| 3.1945 - 3.1946 \right|$$

$$= \left| -0.0001 \right|$$

$$= 0.0001$$

And % error is given by

$$\% \ error = \frac{error}{exact \ value} \times 100\%$$

$$= \frac{0.0001}{3.1945} \times 100\%$$

$$= 0.002\%$$

Question #2   **Marks 10**

Evaluate the following double integral,

$$\int_{1}^{2} \int_{1}^{2} \frac{1}{x^3 + y^3} dx dy$$

By using trapezoidal rule, with h = k = 0.25.

Solution:

Taking X = 1, 1.25, 1.50, 1.75, 2.00 and y = 1, 1.25, 1.50, 1.75, 2.00, the following table is obtained using the integrand

$$f(x, y) = \frac{1}{x^3 + y^3}$$

X , Y	1.00	1.25	1.50	1.75	2.00
1.00	0.5	0.3386	0.2286	0.1572	0.1111
1.25	0.3386	0.2560	0.1877	0.1368	0.1005
1.50	0.2286	0.1877	0.1481	0.1145	0.0879
1.75	0.1572	0.1368	0.1145	0.0933	0.0749
2.00	0.1111	0.1005	0.0879	0.0749	0.0625

Now using trapezoidal, keeping variable x fixed mund changing the variable y.

$$\int_{1}^{2} f(1, y) dy = \frac{0.25}{2} [0.5 + 2(0.3386 + 0.2286 + 0.1572) + 0.1111]$$

= 0.2575

$$\int_1^2 f(1.25, y)dy = \frac{0.25}{2}[0.3386 + 2(0.2560 + 0.1877 + 0.1368) + 0.1005]$$

= 0.02

$$\int_1^2 f(1.50, y)dy = \frac{0.25}{2}[0.2286 + 2(0.1877 + 0.1481 + 0.1145) + 0.0879]$$

= 0.1521

$$\int_1^2 f(1.75, y)dy = \frac{0.25}{2}[0.1572 + 2(0.1368 + 0.1145 + 0.0933) + 0.0749]$$

= 0.1152

$$\int_1^2 f(2.00, y)dy = \frac{0.25}{2}[0.1111 + 2(0.1005 + 0.0879 + 0.0749) + 0.0625]$$

= 0.0875

$$\int_1^2 \int_1^2 \frac{1}{x^3 + y^3}dxdy = \frac{h}{2}[f(1, y) + 2\{f(1.25, y) + f(1.50, y) + f(1.75, y)\} + f(2.00, y)]$$

$$= \frac{0.25}{2}[2575 + 2(0.2 + 0.1521 + 0.1152) + 0.0875]$$

= 0.1600

Simpson's 1/3 and 3/8 rules

A Simpson's 1/3 Rule:

Question: Evaluate the following integral, $\int_{1.0}^{1.8} \frac{e^x + e^{-x}}{2}dx$

Using Simpson's 1/3 rule by taking h=0.2 and e=2.7183

Solution:

First of all table of the function is created,

x	$y = f(x) = \dfrac{e^x + e^{-x}}{2}$
1.0	1.543
1.2	1.811
1.4	2.151
1.6	2.577
1.8	3.107

Simpson's $\frac{1}{3}$ rd rule for the given function is

$$I = \frac{h}{3}\left[f_0 + 4(f_1 + f_3) + 2f_2 + f_4\right]$$

$$I = \frac{0.2}{3}\left[1.543 + 4(1.811 + 2.577) + 2(2.151) + 3.107\right]$$

$$I = \frac{0.2}{3}\left[1.543 + 17.552 + 4.302 + 3.107\right]$$

$$I = \frac{0.2}{3}\left[26.504\right] = 1.7669$$

Question 1   **marks 10**

A river is 80 feet wide. Depth d in feet at a distance of x feet from one bank is given by the following table.

Find

x	0	10	20	30	40	50	60	70	80
d	0	4	7	9	12	15	14	8	3

approximately the area of the cross-section by Simpson's 1/3 rule.

SOLUTION

Let $y = d$, depth in feet, then the table is given by

Here

x	0	10	20	30	40	50	60	70	80
$y = d$	0	4	7	9	12	15	14	8	3

$h = 10$

Let A be the area of the cross-section, then the area of the cross-section is given by

$$A = \int_0^{80} y \, dx$$

And the area of cross-section by Simpson's 1/3 rule is given by

$$A = \int_0^{80} y \, dx = \frac{h}{3}[y_0 + 4(y_1 + y_3 + y_5) + 2(y_2 + y_4 + y_6) + y_8]$$

$$= \frac{10}{3}[0 + 4(4 + 9 + 15 + 8) + 2(7 + 12 + 14) + 3]$$

$$= \frac{10}{3}[0 + 4(36) + 2(33) + 3]$$

$$= \frac{10}{3}[0 + 144 + 66 + 3]$$

$$= \frac{10}{3}(213)$$

$$= 710 \, sq.ft$$

A Simpson's 3/8 Rule:
Use Simpson's 3/8 rule to estimate the number of square feet of hand in given lots when x and y are measured in feet   (Use accuracy up to three places of decimal)

x	0	100	200	300	400	500	600	700	800	900	1000
y	125	125	120	112	90	90	95	88	75	35	0

Solution

Simpson's 3/8th rule may be written as

$$I = \frac{3h}{8}[f_0 + 3(f_1 + f_2) + 2f_3 + 3(f_4 + f_5) + 2f_6 + 3(f_7 + f_8) + 2f_9 + f_{10}]$$

$$I = \frac{3(100)}{8}[125 + 3(125 + 120) + 2(112) + 3(90 + 90) + 2(95) + 3(88 + 75) + 2(35) + 0]$$

$$I = \frac{3(100)}{8}[125 + 3(245) + 2(112) + 3(180) + 2(95) + 3(163) + 2(35)]$$

$$I = (37.5)[125 + 735 + 224 + 540 + 190 + 489 + 70]$$

$$I = (37.5)[2375] = 88987.5$$

Differential Equations

Taylor Series Method

Question 1   **marks 10**

Apply Taylor's Series algorithm of order 2 on the initial value problem

$$y' = x^2 + y^2 \; ; \quad y(0) = 1 \; ; \quad h = 0.05$$

in the interval [0,0.1].

Sol.

As $y' = x^2 + y^2$ Since $x_0 = 0$ and $y_0 = 1$

so $y'_0 = x_0^2 + y_0^2 = 0 + 1 = 1$

Now $y_0'' = 2x_0 + 2y_0 y_0' = 2(0) + 2(1)(1) = 2$

As Taylor's Series method algorithm of order 2 is

$$y(t) = y(t_0) + (t - t_0)y'(t_0) + \frac{(t - t_0)^2}{2!}y''(t_0)$$

$$y(t) = y(t_0) + hy'(t_0) + \frac{h^2}{2!}y''(t_0)$$

$$y(.05) = y_1 = 1 + .05(1) + \frac{(.05)^2}{2!}(2) = 1.0525$$

Now we find y_2, so $y'_1 = x_1^2 + y_1^2 = (.05)^2 + (1.0525)^2 = .0025 + 1.1077 = 1.11025$

Now $y_1'' = 2x_1 + 2y_1 y_1 = 2(.05) + 2(1.0525)(1.11025) = .1 + 2.337 = 2.437$

Now $y(t) = y_2 = y(t_1) + hy'(t_1) + \frac{h^2}{2!}y''(t_1)$

$$y(0.1) = y_2 = 1.0525 + (.05)(1.11025) + \frac{(.05)^2}{2!}(2.437)$$

$$= 1.0525 + .0555 + .003 = 1.111$$

Question 2   **marks 10**

Given $y' = 3x^2 + 2y$ and $y(0) = 2$, find by Taylor's series $y(0.1)$ and $y(0.2)$ taking $h = 0.1$.

SOLUTION

First few derivatives from the given differential equation are as follows

$$y' = 3x^2 + 2y$$
$$y'' = 6x + 2y'$$
$$y''' = 6 + 2y''$$
$$y^{(iv)} = 2y'''$$

The initial condition is given as

$$x_0 = 0 , \ y_0 = 2$$

So, using the given initial condition, we have

$$y_0' = 3x_0^2 + 2y_0 = 3(0)^2 + 2(2) = 4$$
$$y_0'' = 6x_0 + 2y_0' = 6(0) + 2(4) = 8$$
$$y_0''' = 6 + 2y_0'' = 6 + 2(8) = 22$$
$$y_0^{(iv)} = 2y_0''' = 2(22) = 44$$
$$y_0^{(v)} = 2y_0^{(iv)} = 2(44) = 88$$

Now, using Taylor's series method up to the fifth term, we have

$$y(x) = y_0 + (x - x_0)y_0' + \frac{(x-x_0)^2}{2}y_0'' + \frac{(x-x_0)^3}{6}y_0'''$$

$$+ \frac{(x-x_0)^4}{24}y_0^{IV} + \frac{(x-x_0)^5}{120}y_0^{V}$$

Substituting the above values of the derivatives, and the initial condition, we obtain

$$y(0.1) = 2 + (0.1 - 0_0)(4) + \frac{(0.1-0)^2}{2}(8) + \frac{(0.1-0)^3}{6}(22)$$

$$+ \frac{(0.1-0)^4}{24}(44) + \frac{(0.1-0)^5}{120}(88)$$

$$y(0.1) = 2 + 0.4 + 0.04 + 0.0037$$
$$+ 0.00018 + 0.00000067$$

$$y(0.1) = 2.44388067 \cong 2.4439$$

So, $y(0.1) = 2.4439$

Now, we have
$$x_1 = 0.1, \ y_1 = 2.4439$$

Using the above condition derivatives are calculated as follows

$$y_1' = 3x_1^2 + 2y_1 = 3(0.1)^2 + 2(2.4439) = 4.9178$$
$$y_1'' = 6x_1 + 2y_1' = 6(0.1) + 2(4.9178) = 10.4356$$
$$y_1''' = 6 + 2y_1'' = 6 + 2(10.4356) = 26.8712$$
$$y_1^{(iv)} = 2y_1''' = 2(26.8712) = 53.7424$$
$$y_1^{(v)} = 2y_1^{(iv)} = 2(53.7424) = 107.4848$$

Substituting the value of y_1 and its derivatives into Taylor's series expansion we get, after retaining terms up to fifth derivative only, we have

$$y(0.2) = y_1 + (x - x_1)y_1' + \frac{(x - x_1)^2}{2}y_1'' + \frac{(x - x_1)^3}{6}y_1'''$$

$$+ \frac{(x - x_1)^4}{24}y_1^{IV} + \frac{(x - x_1)^5}{120}y_1^{V}$$

$$y(0.2) = 2.4439 + (0.2 - 0.1)(4.9178) + \frac{(0.2 - 0.1)^2}{2}(10.4356) + \frac{(0.2 - 0.1)^3}{6}(26.8712)$$

$$+ \frac{(0.2 - 0.1)^4}{24}(53.7424) + \frac{(0.2 - 0.1)^5}{120}(107.4848)$$

$$y(0.2) = 2.4439 + 0.49178 + 0.052178 + 0.004785 + 0.000224 + 0.000009$$

$$= 2.992876 \cong 2.9929$$

So, $y(0.2) = 2.9929$

Euler Method

Question 2   **marks 10**

Use Euler's method to approximate y when x=1, given that

$$\frac{dy}{dx} = \frac{y - x}{y + x}, y(0) = 1, taking\ h = 0.2$$

Sol.

$$\frac{dy}{dx} = \frac{y - x}{y + x}, y(0) = 1, taking\ h = 0.2$$

$$here\ y_{m+1} = y_m + hf(x_m, y_m)$$

$$f(x, y) = \frac{y - x}{y + x}$$

$$x_0 = 0\ ,\ y_0 = 1$$

$$y_1 = y_0 + hf(x_0, y_0)$$

$$= 1 + (0.2)\frac{1 - 0}{1} = 1.2$$

$x_1 = 0.2$, $y_1 = 1.2$

$y_2 = y_1 + hf(x_1, y_1)$

$= 1.2 + (0.2)\dfrac{1.2 - 0.2}{1.2 + 0.2}$ $= 1.2 + (0.2)(0.7142) = 1.2 + 0.14284 = 1.34284$

$x_2 = 0.4$, $y_2 = 1.34284$

$y_3 = y_2 + hf(x_2, y_2)$

$= 1.34284 + (0.2)\dfrac{1.34284 - 0.4}{1.34284 + 0.4}$ $= 1.34284 + (0.2)(0.10819) = 1.34284 + 0.10819 = 1.45103$

$x_3 = 0.6$, $y_3 = 1.45103$

$y_4 = y_3 + hf(x_3, y_3)$

$= 1.45103 + (0.2)\dfrac{1.45103 - 0.6}{1.45103 + 0.6}$ $= 1.45103 + (0.2)(0.40327) = 1.45103 + 0.08065 = 1.531684$

$x_4 = 0.8$, $y_4 = 1.531684$

$y_5 = y_4 + hf(x_3, y_3)$

$= 1.531684 + (0.2)\dfrac{1.531684 - 0.8}{1.531684 + 0.8}$ $= 1.531684 + (0.2)(0.31380) = 1.531684 + 0.06276 = 1.594444$

$x_5 = 1$, $y_5 = 1.594444$

$y_6 = y_5 + hf(x_5, y_5)$

$= 1.594444 + (0.2)\dfrac{1.594444 - 1}{1.594444 + 1}$ $= 1.594444 + (0.2)(0.22912) = 1.594444 + 0.045824 = 1.640268$

Euler Modified Method:

Question 1   **marks 10**

Given that $y' = \log(x + y)$ with $y(0) = 1$, Use Modified Euler's method to find $y(0.2)$, taking $h = 0.2$

SOLUTION

The given differential equation is

$\dfrac{dy}{dx} = y' = \log(x + y), \quad y(0) = 1$
,

So, here $x_0 = 0, \ y_0 = 1$,

$h = 0.2$

At first, we use Euler's method to get

$$y_1^{(1)} = y_0 + hf(x_0, y_0)$$
$$= y_0 + h\log(x_0 + y_0)$$
$$= 1 + 0.2[\log(0+1)]$$
$$= 1 + 0.2\log(1)$$
$$= 1 + 0.2(0)$$
$$= 1 + 0$$
$$= 1$$

So $x_1 = 0.2, \; y_1^{(1)} = 1$

Then, we use modified Euler's method to find

$$y(0.2) = y_1 = y_0 + h\frac{f(x_0, y_0) + f\left(x_1, y_1^{(1)}\right)}{2}$$

$$= 1 + (0.2)\frac{\log(0+1) + \log(0.2+1)}{2}$$

$$= 1 + (0.2)\frac{\log(1) + \log(1.2)}{2}$$

$$= 1 + (0.1)[\log(1) + \log(1.2)]$$

$$= 1 + (0.1)(0 + 0.0792)$$

$$= 1 + 0.1(0.0792)$$

$$= 1 + 0.00792$$

$$= 1.00792$$

Runge-Kutta Method

Question 2   **marks 10**

Given $y' = x + 2y + 1, y(0) = 2$,

Find $y(1)$ and $y(2)$ using the Runge-Kutta method of fourth order taking $h = 1$

SOLUTION

Given differential equation is
$$y' = x + 2y + 1, y(0) = 2$$

Here $h = 1$ and $x_0 = 0, y_0 = 2$

The Fourth-Order Runge-Kutta Method is described as :-

$$y_{n+1} = y_n + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

where,

$$k_1 = hf\left(x_n, y_n\right)$$

$$k_2 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_1}{2}\right)$$

$$k_3 = hf\left(x_n + \frac{h}{2}, y_n + \frac{k_2}{2}\right)$$

$$k_4 = hf\left(x_n + h, y_n + k_3\right)$$

First Iteration:-

$$x_0 = 0 \quad , \quad y_0 = 2$$

$$k_1 = hf(x_0, y_0)$$
$$= 1f(0, 2)$$
$$= 1[0 + 2(2) + 1]$$
$$= 1(5)$$
$$= 5$$

$$k_2 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right)$$
$$= 1f(0 + \frac{1}{2}, 2 + \frac{5}{2})$$
$$= f(0.5, 4.5)$$
$$= 0.5 + 2(4.5) + 1$$
$$= 10.5$$

$$k_3 = hf\left(x_0 + \frac{h}{2}, y_0 + \frac{k_2}{2}\right)$$
$$= 1f(0 + \frac{1}{2}, 2 + \frac{10.5}{2})$$
$$= f(0.5, 7.25)$$
$$= 0.5 + 2(7.25) + 1$$
$$= 16$$

$$k_4 = hf\left(x_0 + h, y_0 + k_3\right)$$
$$= 1f(0 + 1, 2 + 16)$$
$$= f(1, 18)$$
$$= 1 + 2(18) + 1$$
$$= 38$$

Now

$$y_{n+1} = y_n + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

$$y_1 = y_0 + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

$$= 2 + \frac{1}{6}\left(5 + 2(10.5) + 2(16) + 38\right)$$

$$= 2 + \frac{1}{6}(5 + 21 + 32 + 38)$$

$$= 2 + \frac{1}{6}(96)$$

$$= 18$$

<u>Second Iteration:-</u>

$$x_1 = 1, \ y_1 = 18, h = 1$$
$$k_1 = hf(x_1, y_1)$$
$$= 1f(1,18)$$
$$= 1[1 + 2(18) + 1]$$
$$= 1(38)$$
$$= 38$$

$$k_2 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right)$$

$$= 1f(1 + \frac{1}{2}, 18 + \frac{38}{2})$$

$$= f(1.5, 37)$$
$$= 1.5 + 2(37) + 1$$
$$= 76.5$$

$$k_3 = hf\left(x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right)$$

$$= 1f(1 + \frac{1}{2}, 18 + \frac{76.5}{2})$$

$$= f(1.5, 56.25)$$
$$= 1.5 + 2(56.25) + 1$$
$$= 115$$

$$k_4 = hf\left(x_1 + h, y_1 + k_3\right)$$
$$= 1f(1 + 1, 18 + 115)$$
$$= f(2, 133)$$
$$= 2 + 2(133) + 1$$
$$= 269$$

So,

$$y_{n+1} = y_n + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

$$y_2 = y_1 + \frac{1}{6}\left(k_1 + 2k_2 + 2k_3 + k_4\right)$$

$$= 18 + \frac{1}{6}\left(38 + 2(76.5) + 2(115) + 269\right)$$

$$= 18 + \frac{1}{6}(38 + 153 + 230 + 269)$$

$$= 18 + \frac{1}{6}(690)$$

$$= 18 + 115$$

$$= 133$$

So,

$$y(1) = 18$$

$$y(2) = 133$$

Milne's Predictor Corrector Method

Question 2
Marks 10

Find the solution of

$$y' = y(x + y), \quad y(0) = 1$$

Using Milne's P-C method at $x = 0.4$ given that $y(0.1) = 1.11689$, $y(0.2) = 1.27739$ and
$y(0.3) = 1.50412$
Solution:

Here,

$x_0 = 0$, $x_1 = 0.1$, $x_2 = 0.2$, $x_3 = 0.3$, $x_4 = 0.4$

and $y_0 = 1, y_1 = 1.11689, y_2 = 1.27739, y_3 = 1.50412$

$y' = (x + y)y$

$y_1' = (x_1 + y_1)y_1 = (0.1 + 1.11689)(1.11689) = 1.3591323$

$y_2' = (x_2 + y_2)y_2 = (0.2 + 1.27739)(1.27739) = 1.8872032$

$y_3' = (x_3 + y_3)y_3 = (0.2 + 1.50412)(1.50412) = 2.713613$

Now, using Predictor Formula

$y_4 = y_0 + \dfrac{4h}{3}\left(2y_1' - y_2' + 2y_3'\right)$

$y_4 = 1 + \dfrac{4*0.1}{3}\left[2(1.3591323) - 1.8872032 + 2(2.713613)\right]$

$y_4 = 1.8344383$

Adam Moultan's Predictor Corrector Method

Question   **marks 10**

 Using Adam-Moulton Predictor-Corrector Formula find f(0.4) from Ordinary Differential Equation

$$y' = 1 + 2xy \quad ; \quad y(0) = 0 \quad ; \quad h = 0.1$$

with the help of following table.

X	0	0.1	0.2	0.3
Y	0	0.1007	0.2056	0.3199

Solution: Here,

$h = 0.1 \qquad f(x, y) = 1 + 2xy$

$y_0' = 1 + 2x_0 y_0 = 1 + 2(0)(0) = 1$

$y_1' = 1 + 2x_1 y_1 = 1 + 2(0.1)(0.1007) = 1.02014$

$y_2' = 1 + 2x_2 y_2 = 1 + 2(0.2)(0.2056) = 1.08224$

$y_3' = 1 + 2x_3 y_3 = 1 + 2(0.3)(0.3199) = 1.19194$

Now, Using Adam's P-C Pair Formula:-

$y_{n+1} = y_n + \dfrac{h}{24}\left(55y'_n - 59y'_{n-1} + 37y'_{n-2} - 9y'_{n-3}\right)$

Putting the values;

$y_4 = y_3 + \dfrac{h}{24}\left(55y'_3 - 59y'_2 + 37y'_1 - 9y'_0\right)$

$y_4 = 0.3199 + \dfrac{0.1}{24}\left(55(1.19194) - 59(1.08224) + 37(1.02014) - 9(1)\right)$

$y_4 = 0.446773833$

Computing y'_4 for the Corrector Formula;

$y'_4 = 1 + 2x_4 y_4 = 1 + 2(0.4)(0.446773833)$

$y'_4 = 1.3574190664$

Now Applying the Corrector Formula;

$y_{n+1} = y_n + \dfrac{h}{24}\left(9y'_{n+1} + 19y'_n - 5y'_{n-1} + y'_{n-2}\right)$

$y_4 = y_3 + \dfrac{h}{24}\left(9y'_4 + 19y'_3 - 5y'_2 + y'_1\right)$

$y_4 = 0.3199 + \dfrac{0.1}{24}\left(9(1.3574190664) + 19(1.19194) - 5(1.08224) + 1.02014\right)$

$y_4 = 0.446869048$

FAQ updated version.

Question: What is Bracketing method?

Answer: Methods such as bisection method and the false position method of finding roots of a nonlinear equation f(x) = 0 require bracketing of the root by two guesses. Such

methods are called bracketing methods. These methods are always convergent since they are based on reducing the interval between the two guesses to zero in on the root.

Question: What is an Open method?

Answer: In the Newton-Raphson method, the root is not bracketed. Only one initial guess of the root is needed to get the iterative process started to find the root of an equation. Hence, the method falls in the category of open methods.

Question: Explain Muller's method briefly.

Answer: In Muller's method, f (x) = 0 is approximated by a second degree polynomial; that is by a quadratic equation that fits through three points in the vicinity of a root. The roots of this quadratic equation are then approximated to the roots of the equation f (x) 0.This method is iterative in nature and does not require the evaluation of derivatives as in Newton-Raphson method. This method can also be used to determine both real and complex roots of f (x) = 0.

Question: Explain the difference between the linear and non-linear equations.

Answer: Linear Equation An algebraic equation is said to be linear in which each term is either a constant or the product of a constant and the first power of a single variable. One or more variables can be involved in the linear equations. e.g. x+3y+z=0 2x-y+4z=7 etc. Non-Linear Equation An equation is said to Non-Linear equation if it is not linear. Equations involving the power of the variable 2 or higher, transcendental, logarithmic and trigonometric equations etc lie in the category of Non-Linear equations. e.g. x2+5x+3=0 sinx+3y+9=0 xlogx-7x+4y=2 etc.

Question: Explain which value is to be choosed as X0 in N-R method.

Answer: If, for a given function, f(a)*f(b)<0, then any value between a and b inclusive can be chosen as x0.

Question: Define iterative method of solving linear equations with two examples.

Answer: Under iterative methods, the initial approximate solution is assumed to be known and is improved towards the exact solution in an iterative way. We consider Jacobi, Gauss-Seidel and relaxation methods under iterative methods.

Question: Define Pivoting.

Answer: The Gaussian elimination method fails if any one of the pivot elements becomes zero. In such a situation, we rewrite the equations in a different order to avoid zero pivots. Changing the order of equations is called pivoting.

Question: Write the two steps of solving the linear equations using Gaussian Elimination method.

Answer: In this method, the solution to the system of equations is obtained in two stages.
   i)   the given system of equations is reduced to an equivalent upper triangular form using elementary transformations
   ii)   the upper triangular system is solved using back substitution procedure

Question: Describe Gauss-Jordan elimination method briefly

Answer: This method is a variation of Gaussian elimination method. In this method, the elements above and below the diagonal are simultaneously made zero. That is a given system is reduced to an equivalent diagonal form using elementary transformations. Then the solution of the resulting diagonal system is obtained. Sometimes, we normalize the pivot row with respect to the pivot element, before elimination. Partial pivoting is also used whenever the pivot element becomes zero.

Question: **Describe briefly Crout's reduction method.**

Answer: Here the coefficient matrix [A] of the system of equations is decomposed into the product of two matrices [L] and [U], where [L] is a lower-triangular matrix and [U] is an upper-triangular matrix with 1's on its main diagonal.

Question: **Describe briefly the Jacobi's method of solving linear equations.**

Answer: This is an iterative method, where initial approximate solution to a given system of equations is assumed and is improved towards the exact solution in an iterative way.

Question: **What is the difference between Jacobi's method and Gauss Seidal method?**

Answer: The difference between jacobi's method and gauss Seidel method is that in jacobi's method the approximation calculated are used in the next iteration for next approximation but in Gauss-seidel method the new approximation calculated is instantly replaced by the previous one.

Question: **What is the basic idea of Relaxation method?**

Answer: We can improve the solution vector successively by reducing the largest residual to zero at that iteration. This is the basic idea of relaxation method.

Question: **How the fast convergence in the relaxation method is achieved?**

Answer: To achieve the fast convergence of the procedure, we take all terms to one side and then reorder the equations so that the largest negative coefficients in the equations appear on the diagonal.

Question: **Which matrix will have an inverse?**

Answer: Every square non-singular matrix will have an inverse.

Question: **What are the popular methods available for finding the inverse of a matrix?**

Answer: Gauss elimination and Gauss-Jordan methods are popular among many methods available for finding the inverse of a matrix.

Question: **Explain Gaussian Elimination Method for finding the inverse of a matrix.**

Answer: In this method, if A is a given matrix, for which we have to find the inverse; at first, we place an identity matrix, whose order is same as that of A, adjacent to A which we call an augmented matrix. Then the inverse of A is computed in two stages. In the first stage, A is converted into an upper triangular form, using Gaussian elimination method In the second stage, the above upper triangular matrix is reduced to an identity matrix by row transformations. All these operations are also performed on the adjacently placed identity matrix. Finally, when A is transformed into an identity matrix, the adjacent matrix gives the inverse of A. In order to increase the accuracy of the result, it is essential to employ partial pivoting

Question: **What are the steps for finding the largest eigen value by power method.**

Answer:
Procedure Step 1: Choose the initial vector such that the largest element is unity.
Step 2: The normalized vector is pre-multiplied by the matrix [A].
 Step 3: The resultant vector is again normalized.

Question: What is the method for finding the eigen value of the least magnitude of the matrix [A]?

Answer: For finding the eigen value of the least magnitude of the matrix [A], we have to apply power method to the inverse of [A].

Question: What is interpolation?

Answer: The process of estimating the value of y, for any intermediate value of x, lying inside the table of values of x is called interpolation.

Question: What is extrapolation?

Answer: The method of computing the value of y, for a given value of x, lying outside the table of values of x is known as extrapolation.

Question: What happens when shift operator E operates on the function.

Answer: When shift operator E operates on the function it results in the next value of the function.Ef(x)=f(x+h)

Question: What is the basic condition for the data to apply Newton's interpolation methods?

Answer: To apply Newton's interpolation methods, data should be equally spaced.

Question: When is the Newton's forward difference interpolation formula used?

Answer: Newton's forward difference interpolation formula is mainly used for interpolating the values of y near the beginning of a set of tabular values and for extrapolating values of y, a short distance backward from y0.

Question: When is the Newton's forward difference interpolation formula used?

Answer: Newton's forward difference interpolation formula is mainly used for interpolating the values of y near the beginning of a set of tabular values and for extrapolating values of y, a short distance backward from y0.

Question: When the Newton's backward difference interpolation formula is used?

Answer: For interpolating the value of the function y = f (x) near the end of table of values, and to extrapolate value of the function a short distance forward from yn, Newton's backward Interpolation formula is used.

Question: What is the formula for finding the value of p in Newton's forward difference interpolation formula?

Answer: P=(x-x0)/h

Question: What is the formula for finding the value of p in Newton's backward difference interpolation formula?

Answer: P=(x-xn)/h

Question: If the values of the independent variable are not equally spaced then which formula should be used for interpolation?

Answer: If the values of the independent variable are not given at equidistant intervals, then the Lagrange's interpolation formula should be used.

Question: To use Newton's divided difference interpolation formula, what should the values of independent variables be?

Answer: To use Newton's divided difference interpolation formula, the values of independent variables should not be equally spaced.

Question: Which difference formula is symmetric function of its arguments?

Answer: Divided difference formula is symmetric function of its arguments.

Question: Is the interpolating polynomial found by Lagrange's and Newton's divided difference formulae is same?

Answer: Yes. The interpolating polynomial found by Lagrange's and Newton's divided difference formulae is one and the same.

Question: Which formula involves less number of arithmetic operations? Newton or Lagrange's?

Answer: Newton's formula involves less number of arithmetic operations than that of Lagrange's.

Question: When do we need Numerical Methods for differentiation and integration?

Answer: If the function is known and simple, we can easily obtain its derivative (s) or can evaluate its definite integral However, if we do not know the function as such or the function is complicated and is given in a tabular form at a set of points x0,x1,…,xn, we use only numerical methods for differentiation or integration of the given function.

Question: If the value of the independent variable at which the derivative is to be found appears at the beginning of the table of values, then which formula should be used?

Answer: If the value of the independent variable at which the derivative is to be found appears at and near the beginning of the table, it is appropriate to use formulae based on forward differences to find the derivatives.

Question: If the value of the independent variable at which the derivative is to be found occurs at the end of the table of values, then which formula should be used?

Answer: If the value of the independent variable at which the derivative is to be found occurs at the end of the table of values, it is appropriate to use formulae based on backward differences to find the derivatives.

Question: Why we need to use RICHARDSON'S EXTRAPOLATION METHOD?

Answer: To improve the accuracy of the derivative of a function, which is computed by starting with an arbitrarily selected value of h, Richardson's extrapolation method is often employed in practice.

Question: To apply Simpson's 1/3 rule, what should the number of intervals be?

Answer: To apply Simpson's 1/3 rule, the number of intervals must be even.

Question: To apply Simpson's 3/8 rule, what should the number of intervals be?

Answer: To apply Simpson's 3/8 rule, the number of intervals must be multiple of 3.

Question: What is the order of global error in Simpson's 1/3 rule?

Answer: The global error in Simpson's 1/3 rule is of the order of 0(h4).

Question: Is the order of global error in Simpson's 1/3 rule equal to the order of global error in Simpson's 3/8 rule?

Answer: Yes. The order of global error in Simpson's 1/3 rule equal to the order of global error in Simpson's 3/8 rule.

Question: What is the order of global error in Trapezoidal rule?

Answer: The global error in Trapezoidal rule is of the order of 0(h2).

Question: What is the formula for finding the width of the interval?

Answer: Width of the interval, h, is found by the formula h=(b-a)/n

Question: What type of region does the double integration give?

Answer: Double integration gives the area of the rectangular region.

Question: Compare the accuracy of Romberg's integration method to trapezoidal and Simpson's rule.

Answer: Romberg's integration method is more accurate to trapezoidal and Simpson's rule

Question: What is the order of global error in Simpson's 3/8 rule?

Answer: The global error in Simpson's 3/8 rule is of the order of 0(h4).

Question: Which equation models the rate of change of any quantity with respect to another?

Answer: An ordinary differential equation models the rate of change of any quantity with respect to another.

Question: By employing which formula, Adam-Moulton P-C method is derived?

Answer: Adam-Moulton P-C method is derived by employing Newton's backward difference interpolation formula.

Question: What are the commonly used number systems in computers?

Answer: Binary Octal Decimal Hexadecimal

Question: If a system has the base M, then how many different symbols are needed to represent an arbitrary number? Also name those symbols.

Answer: M 0, 1, 2, 3, ... , M-1

Question: What is inherent error and give its example.

Answer: It is that quantity of error which is present in the statement of the problem itself, before finding its solution. It arises due to the simplified assumptions made in the mathematical modeling of a problem. It can also arise when the data is obtained from certain physical measurements of the parameters of the problem. e.g. if writing 8375 instead of 8379 in a statement lies in the category of inherent error.

Question: What is local round-off error?

Answer: At the end of computation of a particular problem, the final results in the computer, which is obviously in binary form, should be converted into decimal form-a form understandable to the user-before their print out. Therefore, an additional error is committed at this stage too. This error is called local round-off error.

Question: What is meant by local truncation error?

Answer: Retaining the first few terms of the series up to some fixed terms and truncating the remaining terms arise to local truncation error.

Question: What is transcendental equation and give two examples.

Answer: An equation is said to be transcendental equation if it has logarithmic, trigonometric and exponential function or combination of all these three. For example it is a transcendental equation as it has an exponential function These all are the examples of transcendental equation.

Question: What is meant by intermediate value property?

Answer: If f(x) is a real valued continuous function in the closed interval if f(a) and f(b) have opposite signs once; that is f(x)=0 has at least one root such that Simply If f(x)=0 is a polynomial equation and if f(a) and f(b) are of different signs ,then f(x)=0 must have at least one real root between a and b.

Question: What is direct methods of solving equations?

Answer: Those methods which do not require any information about the initial approximation of root to start the solution are known as direct methods. The examples of

direct methods are Graefee root squaring method, Gauss elimination method and Gauss Jordan method. All these methods do not require any type of initial approximation.

Question: What is Iterative method of solving equations?

Answer: These methods require an initial approximation to start. Bisection method, Newton raphson method, secant method, jacobi method are all examples of iterative methods

Question: If an equation is a transcendental, then in which mode the calculations should be done?

Answer: All the calculations in the transcendental equations should be done in the radian mode.

Question: What is the convergence criterion in method of iteration?

Answer: If be a root of f(x) =0 which is equivalent to I be any interval containing the point x= and will converge to the root provided that the initial approximation is chosen in I

Question: When we stop doing iterations when Toll is given?

Answer: Here if TOL is given then we can simply find the value of TOL by subtracting both the consecutive roots and write it in the exponential notation if the required TOL is obtained then we stop.

Question: How the value of h is calculated in interpolation?

Answer: There are two types of data in the interpolation one is equally spaced and other is unequally spaced. In equally spaced data we need to calculate the value of h that is calculated by subtracting any two consecutive values and taking their absolute value.

Question: What is an algebraic equation?

Answer: An algebraic equation is an equation which is purely polynomial in any variable. Supposed x2+3x+2=0, x4+3x2=0, y3+6y2=0 all are algebraic equations as these are purely polynomial in x and y variable.

Question: What is Descartes rule of signs?

Answer: The number of positive roots of an algebraic equation f(x) =0 can not exceed the no of changes in signs. Similarly the no of negative roots of negative roots of and algebraic equation can not exceed the no of changes in sign of equation f (-x) =0.

Question: What are direct methods?

Answer: The numerical methods which need no information about the initial approximation are known as direct methods like Graffee's root squaring method.

Question: What is meant by iterative methods?

Answer: The methods which need one or more iterations are known as iterative methods like bisection method, Newton raphson method, and many other methods.

Question: What is graphically meant by the root of the equation?

Answer: If the graph of a function f(x) =0 cuts the x-axis at a point a then a is known as the root of the equation.

Question: Q.   What is the difference between open and bracketing method?

Answer: In open methods we need only one initial approximation of the root that may be any where lying and if it is not very close then we have to perform more iteration and the example of open method is Newton raphson method. In bracketing method we bracket the root and find that interval in which root lies means we need two initial

approximations for the root finding. Bisection method is an example of the bracketing method.

Question: Condition for the existence of solution of the system of equations.

Answer: If the |A| is not equal to zero then the system will have a unique solution if |A|=0 then the system will have no solution

Question: Should the system be diagonally dominant for gauss elimination method?

Answer: The system of equation need not to be diagonally dominant for Gauss elimination method and gauss Jordan method for both the direct method it is not necessary for the system to be diagonally dominant .it should be diagonally dominant for iterative methods like jacobie and gauss seidel method.

Question: What is meant by diagonally dominant system?

Answer: A system a1x+b1y+c1z=d1, a2x+b2y+c2z=d2, a3x+b3y+c3z=d3 is said to be diagonally dominant if the following condition holds. |a1| => (greater or equal) |b1|+|c1| |b2| => (greater or equal) |a2|+|c2| |c3| => (greater or equal) |a3|+|b3|

Question: State the sufficient condition for the convergence of the system of equation by iterative methods.

Answer: A sufficient condition for convergence of iterative solution to exact solution is |a1| => (greater or equal) |b1|+|c1| |b2| => (greater or equal) |a2|+|c2| |c3| => (greater or equal) |a3|+|b3| For the system a1x+b1y+c1z=d1, a2x+b2y+c2z=d2, a3x+b3y+c3z=d3 Similarly for the system with more variables we can also construct the same condition

Question: The calculation for numerical analysis should be done in degree or radians.

Answer: All the calculation for numerical analysis should be done in radians not in degrees set your calculator in radians mode and suppose the value of pi=3.14.

Question: How we can identify that Newton forward or backwards interpolation formula is to be used.

Answer: If the value at which we have to interpolate is in the start of the table then we will use Newton's forward interpolation formula if it is at the end of the table then we will use the Newton's backward interpolation formula.

Question: What is meant precision and accuracy?

Answer: Precision and accuracy are two terms which are used in numerical calculations by precision we mean that how the values in different iterations agree to each other or how close are the different values in successive iterations. For example you have performed 3 different iterations and result of all the iteration are 1.32514,1.32516,31.32518 these three values are very precise as these values agree with each other . Accuracy means the closeness to the actual value. Suppose that you have calculated an answer after some iteration and the answer is 2.718245 and the actual answer is 2.718254 and the answer calculated is very accurate but if this answer is 2.72125 then it is not accurate.

Question: What is the condition that a root will lie in an interval.

Answer: Suppose that you have a function f(x) and an interval [a,b]and you calculate both f(a)and f(b)if f(a)f(b)<0 then there must exist a root for this function in this interval.In simple words f(a) and f(b)must have opposite signs.

Question: How the divided difference table is constructed?

Answer: x y First difference Second difference Third difference 1 0.2 y0 y1-y0/x1-x0=sy0 sy1- sy0/x2-x0=s2y0 s2y0-s2y1/x3-x0=s3y0 s3y1-s3y0/x4x0 =s4y0 2 0.69 y1 y2-y1/x2-x1=sy1 sy2- sy1/x3-x1=s2y1 s2y2-s2y1/x4-x1=s3y1 3 0.985 y2 y3-y2/x3-x2=sy2 sy3- sy2/x4-x1=s2y2 4 1.2365 y3 y4-y3/x4-x3=sy3 5 2.3651 y4 This is the complete table showing the central differences . Here s stands for difference

Question: What is Gauss-seidel Method

Answer: It is also an iterative method and in this we also check either the system is diagonally dominant and, if so then we proceed on the same lines as in the Jacobi's method. The difference in both is when we calculate a value of any variable that is instantly replaced by the previous value. If we have initial values like x1=0,x2=0 and x3=0 in the case of three variables in the first iteration we will use all the three values to calculate x1 but to calculate the value of x2 we will use the recent value of x1 which is calculated in the first iteration.

Question: What is partial and full pivoting?

Answer: Partial and full pivoting, In gauss elimination method when you have any of diagonal element aii zero it means the solution does not exist to avoid this we change the equation so that a non zero pivot is achieved .Now you have an argument matrix in which he elements in the first column are 1,3,4 respectively here in case of partial pivoting we will replace first element with the last element it is done by replacing the first equation with last equation it is known as partial pivoting. In full pivoting we change rows and columns but that is not implemented manually it is used in computers.

Question: How the initial vector is choose in power method?

Answer: Choose the initial vector such that the largest element is unity. But no problem with this you will precede with this vector as it is and will not change it. In the given question the initial vector is provided so you have to proceed with the given vector and will normalize this vector and normalizing this vector you will keep in mind that the greatest value should be unity. An eigenvector Vis said to be normalized if the coordinate of largest magnitude is equal to unity Actually this condition is for the normalization of the vectors when you have to normalize the vector you keep in mind that the largest entery in the vector must be 1 and so you take the largest element and divide the remaining values by the greatest value.

Question: what is the relation ship between p=0 and non zero p in interpolation.

Answer: fp=fo+pDfo+1/2(p2-p)D2fo+1/6(p3-3p2+2p)D3fo+1/24(p4-6p3+11p2-6p)D4fo+... This is the interpolation formula For the derivative you will have to take the derivative of this formula w.r.t p and you will get f'p=1/h{Dfo+1/2(2p-1)D2fo+1/6(3p2-6p+2)D3fo+...} Now put here p=0 f'p=1/h{Dfo+1/2(0-1)D2fo+1/6(0-0+2)D3fo+...} So the formula becomes f'p=1/h{Dfo+D2fo/2+D3fo/3+...} so this is the relation ship between non zero and zero p no when you have to calculate p the you use formula p=x-xo/h so this is the impact of x.

Question: What is chopping and Rounding off?

Answer: Chopping and rounding are two different techniques used to truncate the terms needed according to your accuracy needs. In chopping you simply use the mentioned number of digits after the decimal Mnd discard all the remaining terms. Explanation (1/3 - 3/11) + 3/20=(0.333333...-0.27272727..)+0.15=(0.333-0.272)+0.15 This is the three digit chopping.

Question: When the forward and backward interpolation formulae are used?

Answer: In interpolation if we have at the start then we use the forward difference formula and the formula to calculate p is x-x0/h. If the value of x lies at the end then we use Newton's backward formula and formula to calculate the value of p is x-xn/h. Now I come to your question as in this case the value lies at the end so 6 will be used as the xn. This procedure has been followed by the teacher in the lectures. But some authors also use another technique that is if you calculate the value of p and that is negative then the origin is shifted to that value for which the value of p becomes positive. And then according to that origin the values of differences are used and you need not follow this procedure.

Question: What is forward and backward difference operator and the construction of their table.

Answer: For forward Dfr =fr+1 –fr Df0 = f1-f0 In terms of y Dyr+1=yr+1-yr D stands for the forward difference operator For backward Dfr =fr –fr-1 Df1 = f1-f0 In terms of y Dy1=y1-y0 Here D stands for backwards operator Now the construction of the difference table is based on X   Y   1st forward 2nd forward 3rd forward x1   Y1   Y2-Y1=Dy0   x2   Y2   Y3-Y2=Dy1   x3   Y3   Y4-Y3=Dy2   x4   Y4   Now consider the construction of table for the backward table X   Y   1st forward 2nd forward 3rd forward x1   Y1 Y2-Y1=Dy1   x2   Y2   Y3-Y2=Dy2   x3   Y3   Y4-Y3=Dy3   x4   Y4
	Dear student this is the main difference in the construction of the forward and backwards difference table when you proceed for forward difference table you get in first difference the value Dy0 but in the construction of backwards difference table in the first difference you get Dy1 and in the second difference in the forward difference table you get D2 y0 and in the backward difference table the first value in the second difference is D2 y1. I think so you have made it clear.

Question: What is Jacobi's method?

Answer: Jacobi's Method It is an iterative method and in this method we first of all check either the system is diagonally dominant and, if the system is diagonally dominant then we will calculate the value of first variable from first equation in the form of other variables and from the second equation the value of second variable in the form of other variables and so on. We are provided with the initial approximations and these approximations are used in the first iteration to calculate first approximation of all the variables. The approximations calculated in the first iteration are used in the second iteration to calculate the second approximations and so on.

Question: what is Simpson's 3/8th rule.

Answer: The general formula for Simpson's 3/8th rule is 3h/8[f0+3(f1+f2)+2f3+(3f4+f5)+2f6+...+3(fn-2+fn-1)+fn] Now if we have to calculate the integral by using this rule then we can simply proceed just write first and last vale and distribute all the remaining values with prefix 3 and 2 Like you have f0,f1,f2,f3,f4 Then the integral can be calculated as 3h/8[f0+f4+3(f1+f2)+2f4] If we have values like f0,f1,f2,f3,f4,f5 Then integral can be calculated as 3h/8[f0+3(f1+f2)+2f3+3f4+f5] Similarly proceeding in this fashion we can calculate the integral in this fashion

Question: what is classic runge-kutta method

Answer: The fourth order Runge-Kutta method is known as the classic formula of classic Runge-Kutta method. yn+1=yn+1/6(k1+2k2+2k3+k4) Where k1=hf(xn,yn) k2=hf(xn+h/2+yn+k1/2) k3=hf(xn+h/2,yn+k'2/2) k4=hf(xn+h,yn+k3)

Question: What is meant by TOL?

Answer: The TOL means the extent of accuracy which is needed for the solution. If you need the accuracy to two places of decimal then the TOL will be 10-2 .similarly the 10-3 means that the accuracy needed to three places of decimal. Suppose you have the root from last iteration 0.8324516 and 0.8324525 if we subtract both and consider absolute value of the difference 0.0000009 now it can be written as 0.09*10-5 so the TOL in this case is 10-5.similarly if we have been provided that you have to for the TOL 10-2 you will check in the same way. In the given equation you will solve the equation by any method and will consider some specific TOL and try to go to that TOL. Some time no TOL is provided and you are asked to perform to some specific no of iterations.

Question: what is meant by uniqueness of LU method.

Answer: An invertible (whose inverse exists) matrix can have LU factorization if and only if all its principal minors(the determinant of a smaller matrix in a matrix) are non zero .The factorization is unique if we require that the diagonal of L or U must have 1's.the matrix has a unique LDU factorization under these condition . If the matrix is singular (inverse does not exists) then an LU factorization may still exist, a square matrix of rank (the rank of a matrix in a field is the maximal no of rows or column) k has an LU factorization if the first k principal minors are non zero. These are the conditions for the uniqueness of the LU decomposition.

Question: how the valu of h is calculated from equally spaced data.

Answer: Consider the following data x y 1 1.6543 2 1.6984 3 2.4546 4 2.9732 5 3.2564 6 3.8765 Here for h=2-1=3-2=1 x y 0.1 1.6543 0.2 1.6984 0.3 2.4546 0.4 2.9732 0.5 3.2564 0.6 3.8765 Here for the calculation of h =0.2-0.1=0.3-0.2=0.1 I think so that you may be able to understand .

Glossary (Updated Version)

Absolute Error :

The absolute error is used to denote the actual value of a quantity less it's rounded value if x and x* are respectively the rounded and actual values of a quantity, then absolute error is defined by AE=|x-x*|

Accuracy :

The Extent of the closeness between the actual value and estimated value is known as accuracy. Suppose you have taken readings like 2.1234,2.1236and 2.1238 and 2.45,2.52,2.63 Now if the actual root is 2.65 then the second values are more accurate and not precise but first set of values is precise but not accurate as it differs from the actual value.

Algebraic equation :

The equation f(x)=0 is known as algebraic equation if it is purely a polynomial in x. Like f(x)=3 ,f(x)=x³-3x-4 are some examples of algebraic equations and the if variable is changed to y x or any one then under the same conditions it will be algebraic.

Bisection method :

It is a bracketing method which is used to locate the root of an equation and it make use of intermediate value property to locate the interval in which the root lies, formula which is used is (a+b)/2 where a and b are points such that f(a)f(b)<0.

Bracketing Method :

The iterative methods which require two initial approximation for it's first iteration are known as bracketing methods. Bisection method is the example of the bracketing method as it requires a interval for the approximation of the root.

Crout's Method :

This method is used to solve the system of the equations by decomposing the system into two matrices L and U where L is a lower triangular matrix and U is an upper triangular matrix. In U all the elements in the main diagonal are 1.

Descartes rule of signs : The no of positive roots of an algebraic equation f(x)=0 with real coefficients cannot exceed the no of changes in the in sign of the coefficients in the polynomial f(x)=0,similarly the no of negative roots of f(x)=0 can not exceed the no of changes in the sign of the coefficients of f(-x)=0

Direct methods :

These are the methods which do not need the knowledge of the initial approximation are known as direct methods.

Gauss Elimination Method :

It is a direct method which is used to solve a system of equations

Gauss Seidel iterative method :

It is an iterative method in which an initial approximation is given. First of all system should be checked either it is diagonally dominant or not if it is not then it is made diagonally dominant. Secondly the first variable is calculated in terms of other variables from first equation and Second variable from second equation and so on .the previous value of the variable is replaced by new value instantly as it is obtained. This is the difference in the gauss seidel and jacobie iterative method.

Graeffee's root squaring method :

This method is used to find all the roots of the polynomial equation.

Intermediate value property :

If for an equation f(x)=0 for two values a and b we have such that f(a)f(b)<0 then there must exist a root between a and b in the interval [a,b].

Inverse of a matrix :

The matrix B is said to be inverse of a matrix A if the product of and B is I identity matrix.

Iterative methods :

Iterative methods are those type of methods which always require an initial approximation to start an iteration

Jacobie's iterative method: :

It is an iterative method in which an initial approximation is given. First of all system should be checked either it is diagonally dominant or not if it is not then it is made diagonally dominant. Secondly the first variable is calculated in terms of other variables from first equation and Second variable from second equation and so on after first iteration in the second iteration all the variables are replaced by the previous one.

Muller's Method :

In Muller's method f(x)=0 is approximated by a second degree polynomial, that is by quadratic equation that fits through three points in the vicinity of a root .Then roots of this quadratic equation are then approximated to the roots of equation f(x)=0

Newton Raphson Method. :

It is an open method which is used to locate a root of the equation, it needs only one initial approximation for it's first iteration. xn+1=xn-(fxn/f'xn)

Non singular matrix :

A matrix is said to be non singular if the determinant of the matrix is non zero. |A| is not equal to zero and the inverse of non singular exists.

Open methods :

The methods which require only one initial approximation to start the first iteration, for example the Newton's Raphsom method is known as the open method as it requires only one initial approximation.

Pivoting :

If any of the pivot elements in gauss elimination become zero then this method fails so to Avoid such type of situation equation are rearranged to get rid of zero pivot element, this procedure is known as pivoting.
Polynomial:

An expression of the form $f(x) = a_0 x^n + a_1 x^{n-1} + a_2 x^{n-2} + ... + a_{n-1} x + a_n$ where n is a positive integer and $a_0, a_1, a_2 + ... a_n$ are real constants, such type of expression is called an nth degree polynomial in x if $a_o \neq 0$

Precision :

The extent of closeness of different measurement taken to estimate a certain value. Suppose you have done different iterations to measure the root of an equation and take different values as 2.1234, 2.1236, and 2.1238 these all the values are very close to each other so these are very precise.

Regula –Falsi method :

It is also an iterative method and is a bracketing method and use intermediate value property to get it's initial guess. The formula used for this is xn+1=xn-(xn-xn-1/f(xn)-f(xn-1))
Relative Error :   It is the ratio of the absolute error to the actual value of the quantity. Thus RE=AE=AE\|x*|

Relaxation method :

It is also an iterative method and in this method you solve the system of equation by making the greatest residual to zero.
Root : If you have an equation f(x)=0 then the no a is said to be the root of the equation if f(a)=0 Suppose you have an equation f(x)=x2-4 the 2 is a root of the equation as f(2)=4-4=0

Secant Method :

The secant method is also an open method and it takes two initial values for it's first approximation, the formula used for this is known as xn+1={xn-1f(xn)-xnf(xn-1)}/{f(xn)-f(xn-1)}

Significant digits :

A significant digit in an approximate no is a digit, which gives reliable information about the size of the number. In other words a significant digit is used to express accuracy, that Is how many digits in the no have meaning.

Singular matrix :

A matrix s said to be a singular matrix if the determinant of the matrix is zero |A|=0,the inverse of the singular matrix do not exist.

Transcendental equation :

An equation f(x) =0 is said to be transcendental equation if it contains trigonometric, logarithmic and exponential functions

Truncation Error :

It is defined as the replacement of one series by another series with fewer terms. The error arising by this approximation is known as truncation error.

Important Formula For MTH603

Bisection Method

$$x2 = \frac{x0 + x1}{2}$$

Muller Method

$$x = \frac{x0 - 2c}{+b - \sqrt{b^2 - 4ac}}$$

in this formula x0,x1, x2 will be and U will just put the values according to the given formulas.

$$a = \frac{h2f1 - (h1 + h2)f0 + h1f2}{h1h2(h1 + h2)}$$

$$b = \frac{f1 = f0 - ah1^2}{h1}$$

$$c = f(x0)$$

$$h1 = x1 - x0$$

$$h2 = x0 - x2$$

Regula Falsi Method (Method of False position)

$$x3 = x2 - \frac{x2 - x1}{f(x2) - f(x1)}f(x2)$$

Newton Rophson method

$$x1 = x0 - \frac{f(x0)}{f'(x0)}$$

Secant Method

$$x2 = \frac{x0 f(x1) - x1 f(x0)}{f(x1) - f(x0)}$$

Newton's Formula

$$x1 = \frac{1}{2}(x0 + n/x0)$$

in this formula
x0= 2 perfect square near to 12 such like 9 and 16

Graffee root squaring method

$$f(x)f(-x) = a_3^2 x^6 - (a_2^2 - 2a_1 a_3)x^4 + (a_1^2 - 2a_0 a_2)x^2 - x_0^2$$

the truncation error (TE) is given by

$$\text{TE} \leq \frac{x^{2n+2}}{(2n+2)!}$$

The TE is independent of the computer used.

Short Questions Paper

Set-01

Approximate the integral $\int_{1}^{1.6} \frac{2x}{x^2+1} dx$ using Simpson's 1/3 rule and calculate the percentage error. (Take result up to 4 decimal places)

Note: In order to get full marks do all necessary steps.

Construct a forward difference table for the following values

x	0.1	0.3	0.5	0.7	0.9	1.1	1.3
y	0.003	0.067	0.148	0.248	0.37	0.518	0.697

Note : In order to get full marks do all necessary steps.

Solve the system

$$4x + 3y = 24$$
$$3x + 4y - z = 30$$
$$-y + 4z = -24$$

by Gauss Seidal Method, taking $(0, 0, 0)^t$ as initial approximation(Two iterations only and take result up to 4 decimal places)

Note : In order to get full marks do all necessary steps

Let $f(x) = x + \dfrac{2}{x}$, use cubic Lagrange interpolation based on the nodes $x_0 = 0.5, x_1 = 1, x_2 = 2 \ and \ x_3 = 2.5$ to approximate $f(1.5)$ and $f(1.3)$.

Note : In order to get full marks do all necessary stepsSolution

Approximate the Dominant Eigenvalue and corresponding Eigenvector for the matrix

$$\begin{bmatrix} 0 & 11 & -5 \\ -2 & 17 & -7 \\ -4 & 26 & -10 \end{bmatrix}$$

by using Power Method. Start with $X_0 = (1,1,1)^t$. (Five iterations only and take result up to 4 decimal places)

Note : In order to get full marks do all necessary steps

==

Set-02

Question No: 31   (Marks: 2)

_____ If

$F(h) = 256.2354$ and $F(\frac{h}{2}) = 257.1379$, then find $F_1(\frac{h}{2})$ using Richardson's extrapolation limit.

Question No: 32   (Marks: 2)

_____ Evaluate the integral

$$\int_{0}^{\frac{\pi}{2}} (\cos x + 2)dx$$

Using Simpson's 3/8 rule

Take h= $\frac{\pi}{4}$

Question No: 33   (Marks: 2)

_____ Write a general formula for Modified Euler's method of solving the given differential equation.

Question No: 34   (Marks: 3)

_____ Evaluate the integral

$$\int_{0}^{4} x^2 dx$$

Using Trapezoidal rule
Take h=1

Question No: 35   (Marks: 3)

_____ Evaluate the integral

$$\int_{3}^{5} (\log x + 2)dx$$

Using Simpson's 3/8 rule
Take h=1

Question No: 36   (Marks: 3)

Write a formula for finding the value of k_3 in Fourth-order R-K method.

Question No: 37   (Marks: 5)

Find Newton's forward difference table from the following data.

x	0.0	0.1	0.2	0.3	0.4
$f(x)$	1	0.9048	0.8187	0.7408	0.6703

Question No: 38   (Marks: 5)

Evaluate the integral

$$\int_0^3 (x^2 + x)dx$$

Using Simpson's 3/8 rule

Take h=1

Question No: 39   (Marks: 5)

Use Runge-Kutta Method of order four to find the values of $k_1, k_2, k_3 \ and \ k_4$ for the initial value problem

$$y' = \frac{1}{2}(2x^3 + y), y(1) = 2$$ taking $h = 0.1$

==

Set-03

Question:

Answer:

==

Multiple Choice Question

Set-01

Question No: 1   (Marks: 1)   - Please choose one

Symbol used for forward differences is

► ∇

► Δ

► δ

► μ

Question No: 2   (Marks: 1)   - Please choose one

_____ The relationship between central difference operator and the shift operator is given by

► $\delta = E - E^{-1}$

► $\delta = E + E^{-1}$

► $\delta = E^{\frac{1}{2}} + E^{-\frac{1}{2}}$

► $\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$

Question No: 3   (Marks: 1)   - Please choose one

_____ Muller's method requires --------starting points

► 1

► 2

► **3**

► 4

Question No: 4   (Marks: 1)   - Please choose one

If S is an identity   matrix, then

► $S^{-1} = S$

► $S^t = S$

► *All are true*

► $S^{-1} = S^t$

Question No: 5   (Marks: 1)   - Please choose one

If we retain r+1 terms in Newton's forward difference formula, we obtain a polynomial of degree ---- agreeing with y_x at $x_{0,} x_1, ..., x_r$

► r+2

► r+1

► **r**

► r-1

Question No: 6   (Marks: 1)   - Please choose one

P in Newton's forward difference formula is defined as

► $p = (\dfrac{x - x_0}{h})$

► $p = (\dfrac{x + x_0}{h})$

$$p = (\frac{x + x_n}{h})$$

►

$$p = (\frac{x - x_n}{h})$$

►

Question No: 7   (Marks: 1)   - Please choose one

_____ Octal

number system has the base ---------------

► 2

► **8**

► 10

► 16

Question No: 8   (Marks: 1)   - Please choose one

_____ Newton's

divided difference interpolation formula is used when the values of the independent variable are

► Equally spaced

► Not equally spaced

► Constant

► **None of the above**

Question No: 9   (Marks: 1)   - Please choose one

_____ Given the

following data

x	0	1	2	4
$f(x)$	1	1	2	5

Value of $f(2,4)$ is

► 1.5

► 3

► 2

► 1

Question No: 10   (Marks: 1)   - Please choose one

If $y(x)$ is approximated by a polynomial $p_n(x)$ of degree n then the error is given by

► $\varepsilon(x) = y(x) + P_n(x)$

► $\varepsilon(x) = y(x) - P_n(x)$

► $\varepsilon(x) = P_n(x) - y(x)$

► $\varepsilon(x) = y(x) \times P_n(x)$

Question No: 11   (Marks: 1)   - Please choose one

Let I denotes the closed interval spanned by $x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, \overline{x}$. Then $F(x)$ vanishes ------times in the interval I.

► n-1

► n+2

► n

► n+1

Question No: 12   (Marks: 1)   - Please choose one

_____ Differential

operator in terms of forward difference operator is given by

$$D = \frac{1}{h}(\Delta + \frac{\Delta^2}{2!} + \frac{\Delta^3}{3!} + \frac{\Delta^4}{4!} + \frac{\Delta^5}{5!} + ...)$$

►

$$D = \frac{1}{h}(\Delta + \frac{\Delta^2}{2} + \frac{\Delta^3}{3} + \frac{\Delta^4}{4} + \frac{\Delta^5}{5} + ...)$$

►

► $$D = \frac{1}{h}(\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} + \frac{\Delta^5}{5} - ...)$$

$$D = \frac{1}{h}(\Delta - \frac{\Delta^2}{2!} + \frac{\Delta^3}{3!} - \frac{\Delta^4}{4!} + \frac{\Delta^5}{5!} - ...)$$

►

Question No: 13   (Marks: 1)   - Please choose one

_____ Finding the first

derivative of $f(x)$ at x =0.4 from the following table:

x	0.1	0.2	0.3	0.4
$f(x)$	1.10517	1.22140	1.34986	1.49182

Differential operator in terms of ----------------will be used.

► Forward difference operator
► **Backward difference operator**
► Central difference operator

► None of the given choices

Question No: 14   (Marks: 1)   - Please choose one

For the given table of values

x	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

$f'(0.1)$, using two-point equation will be calculated as.............

► -0.5

► 0.5

► 0.75

► **-0.75**

Question No: 15   (Marks: 1)   - Please choose one

In Simpson's 1/3 rule, $f(x)$ is of the form

► $ax+b$

► ax^2+bx+c

► $ax^3 + bx^2 + cx + d$

► $ax^4 + bx^3 + cx^2 + dx + e$

Question No: 16   (Marks: 1)   - Please choose one

While integrating

$$I = \int_a^b f(x)dx$$

, h , width of the interval, is found by the formula-----.

► $\dfrac{b-a}{n}$

► $\dfrac{b+a}{n}$

► $\dfrac{a-b}{n}$

► None of the given choices

Question No: 17   (Marks: 1)   - Please choose one

To apply Simpson's 1/3 rule, valid number of intervals are.....

► 7

► 8

► **5**

► 3

Question No: 18   (Marks: 1)   - Please choose one

For the given table of values

x	02	0.3	0.4	0.5	0.6	0.7
$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

$f''(0.2)$
, using three-point equation will be calculated as ……

► 17.5

► 12.5

► **7.5**

► -12.5

Question No: 19   (Marks: 1)   - Please choose one

To apply Simpson's 1/3 rule, the number of intervals in the following must be

► 2

► 3

► **5**

► 7

Question No: 20   (Marks: 1)   - Please choose one

To apply Simpson's 3/8 rule, the number of intervals in the following must be

► 10

► 11

► **12**

► 13

Question No: 21   (Marks: 1)   - Please choose one

If the root of the given equation lies between a and b, then the first approximation to the root of the equation by bisection method is ……

► $\dfrac{(a+b)}{2}$

► $\dfrac{(a-b)}{2}$

► $\dfrac{(b-a)}{2}$

► None of the given choices

Question No: 22   (Marks: 1)   - Please choose one

............lies in the category of iterative method.

► Bisection Method

► Regula Falsi Method

► Secant Method

► None of the given choices

Question No: 23   (Marks: 1)   - Please choose one

For the equation $x^3 + 3x - 1 = 0$, the root of the equation lies in the interval......

► (1, 3)

► (1, 2)

► (0, 1)

► (1, 2)

Question No: 24   (Marks: 1)   - Please choose one

_____ Rate of change of any quantity with respect to another can be modeled by

► An ordinary differential equation

► A partial differential equation

► A polynomial equation

► None of the given choices

Question No: 25   (Marks: 1)   - Please choose one

_____ If

$$\frac{dy}{dx} = f(x, y)$$

Then the integral of this equation is a curve in

► None of the given choices

► xt-plane

► yt-plane

► xy-plane

Question No: 26   (Marks: 1)   - Please choose one

In solving the differential equation

$$y' = x + y \; ; \; y(0.1) = 1.1$$

$h = 0.1$, By Euler's method $y(0.2)$ is calculated as

► 1.44

► 1.11

► 1.22

► 1.33

Question No: 27   (Marks: 1)   - Please choose one

In second order Runge-Kutta method

k_1 is given by

► $k_1 = hf(x_n, y_n)$

► $k_1 = 2hf(x_n, y_n)$

► $k_1 = 3hf(x_n, y_n)$

► None of the given choices

Question No: 28   (Marks: 1)   - Please choose one

In fourth order Runge-Kutta method, k_2 is given by

$$k_2 = hf(x_n + \frac{h}{2}, y_n + \frac{k_1}{2})$$

►

$$k_2 = hf(x_n + \frac{h}{3}, y_n + \frac{k_1}{3})$$

►

$$k_2 = hf(x_n - \frac{h}{3}, y_n - \frac{k_1}{3})$$

►

$$k_2 = hf(x_n - \frac{h}{2}, y_n - \frac{k_1}{2})$$

►

Question No: 29   (Marks: 1)   - Please choose one

_____ In fourth order

Runge-Kutta method, k_4 is given by

► $$k_3 = hf(x_n + 2h, y_n + 2k_3)$$

► $$k_3 = hf(x_n - h, y_n - k_3)$$

► $$k_3 = hf(x_n + h, y_n + k_3)$$

► None of the given choices

Question No: 30   (Marks: 1)   - Please choose one

_____ Adam-Moulton

P-C method is derived by employing

► Newton's backward difference interpolation formula

► Newton's forward difference interpolation formula

► Newton's divided difference interpolation formula

► None of the given choices

==

Set-02

/wEPDwUKMTY2I

/wEWBgK+3MqI/

Question # 1 of 8 (**Start time: 01:24:49 PM**)   **Total Marks:** 1

The need of numerical integration arises for evaluating the definite integral of a function that has no explicit _____ or whose antiderivative is not easy to obtain.

► **Select correct option:**

○ antiderivative

○ derivatives

           ►   Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgLy4ZCpA

Time Left **59** sec(s)

Question # 2 of 8 (**Start time: 01:26:18 PM**)   **Total Marks:** 1

In Runge – Kutta Method, we do not need to calculate higher order derivatives and find greater accuracy.

► **Select correct option:**

○ TRUE

○ FALSE

▶ Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgLlwO3cC

Time Left **79** sec(s)

Question # 3 of 8 (**Start time: 01:27:35 PM**)   **Total Marks:** 1

An indefinite integral may _____ in the sense that the limit defining it may not exist.

▶ **Select correct option:**

○ diverge

○ converge

▶ Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWCgKMu86bl

Time Left **85** sec(s)

Question # 5 of 8 (**Start time: 01:30:21 PM**)   **Total Marks:** 1

The Trapezoidal Rule is an improvement over using rectangles because we have much less "missing" from our calculations. We used _____ to model the curve in trapezoidal Rule.

▶ **Select correct option:**

○ straight lines

○ curves

○ parabolas

○ constant

▶ Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgK6oIaxB

Time Left **83** sec(s)

Question # 6 of 8 (**Start time: 01:31:18 PM**)   **Total Marks:** 1

An improper integral is the limit of a definite integral as an endpoint of the interval of integration approaches either a specified real number or 8 or -8 or, in some cases, as both endpoints approach limits.

▶ **Select correct option:**

○ TRUE

○ | FALSE |

▶ Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgKPg8yy[

Time Left **84** sec(s)

Question # 7 of 8 (**Start time: 01:32:33 PM**)   **Total Marks:** 1

Euler's Method numerically computes the approximate derivative of a function.

▶ **Select correct option:**

○ | FALSE |

○ | TRUE |

▶ Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWCgKK6eLRI

Time Left **84** sec(s)

Question # 8 of 8 (**Start time: 01:33:57 PM**)   **Total Marks:** 1

If we wanted to find the value of a definite integral with an infinite limit, we can instead replace the infinite limit with a variable, and then take the limit as this variable goes to _____.

▶ **Select correct option:**

○ | constant |

○ | finite |

○ | infinity |

○ | zero |

▶ | Click here to Save Answer & Move to Next Question |

==

Set-03

No.	Questions	Mks
1	Find value of given data by Adam Moultan's method	10
2	Find value of given data by Dividend Difference Composite method	10
3	Draw backward difference tables for given Data	5+5

x			
y			

x			
y			

4	Write Simpson's 1/3 formula	2
5	Find value by Euler's Method	3
6	Find value of K1 by 2nd Order R-K method	2
7	Convergence is used when ----------	1
8	Bisection method is ……………….. method • Bracketing Method • Open • Random • none http://vustudents.ning.com/	1
9	Newton Raphson method is ………………... method • Bracketing Method • Open • Random • none	1
10	Eigenvalue is • Real • Vector • odd • even	1
11	Find value of y'(1) by Euler's Method taking h=1	2
12	Find value of y'(3) from given table. 	x			
y					2
13	Find value of y'(0.3) by Lagrange's Method 	x			
y					3
14	For Simpson's 1/3 rule no.of intervals must be • 1 • 3 • 5 • 8	1

15	For Simpson's 1/3 rule valid no.of intervals are • 1 • 3 • 5 • 8	1
16	For Simpson's 3/8 rule no.of intervals must be • 10 • 11 • 12 • 14 http://vustudents.ning.com/	1
17	Find the value of y'(1) from given forward difference table 	2

x	y	Δy	Δy^2	Δy^3

Numerical analysis mth603 paper

Numerical Analysis numerical paper 2009

The paper was very easy.

The mcq's were really easy.

Most of the mcq's were from the last 5 lecture.

and from jacobi's method and other's.

then the logical mcq's like

s inverse * s = I

it was repeated twice.

one 5 mark question was from newton's rapson method

one 10 mark question was from lecture 11 page 69 example

best of luck

==

Set-04

/w EPDw UKMTY2

/w EWBgKPm46n

Time Left **89** sec(s)

Quiz Start Time: 04:40 PM

Question # 1 of 10 (Start time: 04:40:08 PM)   Total Marks: 1

The determinant of a diagonal matrix is the product of the diagonal elements.

Select correct option:

○ TRUE

○ FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWBgKin4OcE

Time Left **89** sec(s)

Quiz Start Time: 04:40 PM

Question # 2 of 10 (Start time: 04:40:58 PM)   Total Marks: 1

Power method is applicable if the eigen vectors corresponding to eigen values are linearly independent.

Select correct option:

○ TRUE

○ FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWBgKx743vl

Time Left **89** sec(s)

Quiz Start Time: 04:40 PM

Question # 3 of 10 (Start time: 04:41:55 PM)   Total Marks: 1

A 3 x 3 identity matrix have three and different eigen values.

Select correct option:

○   TRUE

○   FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWBgKR8tba/

Time Left **89** sec(s)

Quiz Start Time: 04:40 PM

Question # 4 of 10 (Start time: 04:42:30 PM)   Total Marks: 1

If n x n matrices A and B are similar, then they have the different eigenvalues (with the same multiplicities).

Select correct option:

○   TRUE

○   FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWBgKjyL19A

Time Left **89** sec(s)

Quiz Start Time: 04:40 PM

Question # 5 of 10 (Start time: 04:42:58 PM)   Total Marks: 1

The Jacobi's method is a method of solving a matrix equation on a matrix that has _____ zeros along its main diagonal.

▶ Select correct option:

○ | no |

○ | atleast one |

▶ | Click here to Save Answer & Move to Next Question |

| /w EPDw UKMTY: |
| /w EWBgKw w uK |

Time Left   89
             sec(s)

Quiz Start Time: 04:40 PM

Question # 6 of 10 (Start time: 04:43:20 PM)   Total Marks: 1

An eigenvector V is said to be normalized if the coordinate of largest magnitude is equal to _____.

▶ Select correct option:

○ | unity |

○ | zero |

▶ | Click here to Save Answer & Move to Next Question |

| /w EPDw UKMTY: |
| /w EWBgLdv/PBE |

Time Left   55
             sec(s)

Quiz Start Time: 04:40 PM

Question # 7 of 10 (Start time: 04:43:39 PM)   Total Marks: 1

The Gauss-Seidel method is applicable to strictly diagonally dominant or symmetric positive definite matrices A.

▶ Select correct option:

○ TRUE

○ FALSE

▶ Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWCgL0zNjNE

Time Left **90** sec(s)

Quiz Start Time: 04:40 PM

Question # 8 of 10 (Start time: 04:45:08 PM)   Total Marks: 1

The determinant of a _____ matrix is the product of the diagonal elements.

▶ Select correct option:

○ diagonal

○ upper triangular

○ low er triangular

○ scalar

▶ Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWCgKw hsW

Time Left **89**

sec(s)

Quiz Start Time: 04:40 PM

Question # 9 of 10 (Start time: 04:46:35 PM)   Total Marks: 1

Eigenvalues of a symmetric matrix are all _____.

▶ Select correct option:

○ real

○ zero

○ positive

○ negative

▶ Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWBgLX34iv A

Time Left   89
             sec(s)

Quiz Start Time: 04:40 PM

Question # 10 of 10 (Start time: 04:47:33 PM)   Total Marks: 1

The Power method can be used only to find the eigenvalue of A that is largest in absolute value—we call this eigenvalue the dominant eigenvalue of A.

▶ Select correct option:

○ TRUE

○ FALSE

==

Set-05

/w EPDw UKMTY2

/w EWBgKqiMqh0

Time Left **89** sec(s)

Quiz Start Time: 04:50 PM

Question # 1 of 10 (Start time: 04:50:37 PM)   Total Marks: 1

The Jacobi's method is a method of solving a matrix equation on a matrix that has no zeros along its main diagonal.

Select correct option:

○ TRUE

○ FALSE

/w EPDw UKMTY2

/w EWCgLw ndf n

Time Left **90** sec(s)

Quiz Start Time: 04:50 PM

Question # 2 of 10 (Start time: 04:50:51 PM)   Total Marks: 1

The characteristics polynomial of a 3x 3 identity matrix is _____, if x is the eigen values of the given 3 x 3 identity matrix. where symbol ^ shows power.

Select correct option:

○ (x-1)^3

○ (x+1)^3

○ x^3-1

○ x^3+1

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWBgKJ5b6A

Time Left **89** sec(s)

Quiz Start Time: 04:50 PM

Question # 3 of 10 (Start time: 04:51:41 PM)   Total Marks: 1

For differences methods we require the set of values.

► Select correct option:

○ TRUE

○ FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWBgKd3KG(

Time Left 88 sec(s)

Quiz Start Time: 04:50 PM

Question # 4 of 10 (Start time: 04:51:57 PM)   Total Marks: 1

If n x n matrices A and B are similar, then they have the different eigenvalues (with the same multiplicities).

Select correct option:

TRUE

FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWBgLC663Y

Time Left 89 sec(s)

Quiz Start Time: 04:50 PM

Question # 5 of 10 (Start time: 04:52:22 PM)   Total Marks: 1

If x is an eigen value corresponding to eigen value of V of a matrix A. If a is any constant, then x – a is an eigen value corresponding to eigen vector V is an of the matrix A - a I.

Select correct option:

○ TRUE

○ FALSE

▶ Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWBgL71Ojw

Time Left **88** sec(s)

Quiz Start Time: 04:50 PM

Question # 6 of 10 (Start time: 04:53:38 PM)   Total Marks: 1

Central difference method seems to be giving a better approximation, however it requires more computations.

▶ Select correct option:

○ TRUE

○ FALSE

▶ Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWBgLTr6Ggl

Quiz Start Time: 04:50 PM

Question # 7 of 10 (Start time: 04:54:56 PM)   Total Marks: 1

The Power method can be used only to find the eigenvalue of A that is largest in absolute value—we call this eigenvalue the dominant eigenvalue of A.

► Select correct option:

○ | TRUE |

○ | FALSE |

► | Click here to Save Answer & Move to Next Question |

| /w EPDw UKMTY2 |

| /w EWBgKr97ef A |

Quiz Start Time: 04:50 PM

Question # 8 of 10 (Start time: 04:55:31 PM)   Total Marks: 1

Iterative algorithms can be more rapid than direct methods.

► Select correct option:

○ FALSE

○ TRUE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWBgL/r4KyC

Time Left 89 sec(s)

Quiz Start Time: 04:50 PM

Question # 9 of 10 (Start time: 04:55:49 PM)   Total Marks: 1

Central Difference method is the finite difference method.

► Select correct option:

○ TRUE

○ FALSE

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2

/w EWBgLonf2RE

Time Left 89 sec(s)

Quiz Start Time: 04:50 PM

Question # 10 of 10 (Start time: 04:56:20 PM)   Total Marks: 1

A 3 x 3 identity matrix have three and different eigen values.

▶ Select correct option:

○ TRUE

○ FALSE

▶ Click here to Save Answer & Move to Next Question

==

Set-06

/wEPDwUKMTY2I

/wEWBgK+3MqI/

Question # 1 of 8 (**Start time: 01:24:49 PM**)   **Total Marks:** 1

The need of numerical integration arises for evaluating the definite integral of a function that has no explicit _____ or whose antiderivative is not easy to obtain.

▶ **Select correct option:**

○ antiderivative

○ derivatives

▶ Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgLy4ZCpA

Time Left **59** sec(s)

Question # 2 of 8 (**Start time: 01:26:18 PM**)   **Total Marks:** 1

In Runge – Kutta Method, we do not need to calculate higher order derivatives and find greater accuracy.

▶ **Select correct option:**

○ TRUE

○ FALSE

▶ Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgLlwO3cC

Time Left **79** sec(s)

Question # 3 of 8 (**Start time: 01:27:35 PM**)   **Total Marks:** 1

An indefinite integral may _____ in the sense that the limit defining it may not exist.

Select correct option:

○ diverge

○ converge

▶ Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWCgKMu86bl

Time Left **85** sec(s)

Question # 5 of 8 (Start time: 01:30:21 PM)   **Total Marks:** 1

The Trapezoidal Rule is an improvement over using rectangles because we have much less "missing" from our calculations. We used _____ to model the curve in trapezoidal Rule.

Select correct option:

○ straight lines

○ curves

○ parabolas

○ | constant |

```
Click here to Save Answer & Move to Next Question
```

/wEPDwUKMTY2I

/wEWBgK6oIaxB

Time Left **83** sec(s)

Question # 6 of 8 (**Start time: 01:31:18 PM**)   **Total Marks:** 1

An improper integral is the limit of a definite integral as an endpoint of the interval of integration approaches either a specified real number or 8 or -8 or, in some cases, as both endpoints approach limits.

▶ **Select correct option:**

○ | TRUE |

○ | FALSE |

```
Click here to Save Answer & Move to Next Question
```

/wEPDwUKMTY2I

/wEWBgKPg8yyD

Time Left **84** sec(s)

Question # 7 of 8 (**Start time: 01:32:33 PM**)   **Total Marks:** 1

Euler's Method numerically computes the approximate derivative of a function.

▶ **Select correct option:**

○ FALSE

○ TRUE

Click here to <u>S</u>ave Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWCgKK6eLRI

Time Left **84** sec(s)

Question # 8 of 8 (**Start time: 01:33:57 PM**)   **Total Marks:** 1

If we wanted to find the value of a definite integral with an infinite limit, we can instead replace the infinite limit with a variable, and then take the limit as this variable goes to _____.

▶ **Select correct option:**

○ constant

○ finite

○ infinity

○ zero

Click here to Save Answer & Move to Next Question

==

Set-07

Question # 1 of 10 (Start time: 10:22:24 AM) Total Marks: 1

Eigenvalues of a symmetric matrix are all _____.
Select correct option:

>   **real**
>   zero
>   positive
>   negative

Question # 2 of 10 (Start time: 10:23:07 AM) Total Marks: 1

An eigenvector V is said to be normalized if the coordinate of largest magnitude is equal to zero.
Select correct option:

>   TRUE
>   **FALSE**

Question # 3 of 10 (Start time: 10:23:55 AM) Total Marks: 1

Exact solution of 2/3 is not exists.
Select correct option:

>   **TRUE**
>   FALSE

Question # 4 of 10 (Start time: 10:24:53 AM) Total Marks: 1

The Gauss-Seidel method is applicable to strictly diagonally dominant or symmetric _____ definite matrices A.
Select correct option:

>   **positive**
>   negative

Question # 5 of 10 (Start time: 10:26:04 AM) Total Marks: 1

Differences methods find the _____ solution of the system.

Select correct option:

numerical
Analytical

Question # 6 of 10 (Start time: 10:26:49 AM) Total Marks: 1

The characteristics polynomial of a 3x 3 identity matrix is _____, if x is the eigen values of the given 3 x 3 identity matrix. where symbol ^ shows power.
Select correct option:

(x-1)^3
(x+1)^3   **i m not sure about this answer**
x^3-1
x^3+1

Question # 7 of 10 (Start time: 10:28:08 AM) Total Marks: 1

The Power method can be used only to find the eigen value of A that is largest in absolute value—----------we call this eigen value the dominant eigen value of A.
Select correct option:

TRUE
FALSE

Question # 8 of 10 (Start time: 10:29:33 AM) Total Marks: 1

The Jacobi's method is a method of solving a matrix equation on a matrix that has no zeros along its _____.
Select correct option:

main diagonal
last column
last row
first row   **i m not sure about this answer**

Question # 9 of 10 (Start time: 10:30:33 AM) Total Marks: 1

If A is a nxn triangular matrix (upper triangular, lower triangular) or diagonal matrix , the eigenvalues of A are the diagonal entries of A.
Select correct option:

TRUE
FALSE

Question # 10 of 10 (Start time: 10:31:28 AM)   **Total Marks: 1**

A 3 x 3 identity matrix have three and different eigen values.
Select correct option:

      TRUE
      FALSE

===

Set-08

Differences methods find the _____ solution of the system.

🚩 **Select correct option:**

   ○   **numerical**

   ○   Analytical

The Power method can be used only to find the eigenvalue of A that is largest in absolute value—we call this eigenvalue the dominant eigenvalue of A.

🚩 **Select correct option:**

   ○   **TRUE**

   ○   FALSE

If n x n matrices A and B are similar, then they have the different eigenvalues (with the same multiplicities).

🚩 **Select correct option:**

   ○   **TRUE**

   ○   FALSE

Eigenvalues of a _____ matrix are all real.

🚩 **Select correct option:**

   ○   **symmetric**

   ○   antisymmetric

   ○   rectangular

   ○   triangular

By using determinants, we can easily check that the solution of the given system of linear equation exits and it is unique.

🚩 **Select correct option:**

○ **TRUE**

○ FALSE

The dominant eigenvector of a matrix is an eigenvector corresponding to the eigenvalue of largest magnitude (for real numbers, smallest absolute value) of that matrix.

▶ **Select correct option:**

○ **TRUE**

○ FALSE

Eigenvalues of a symmetric matrix are all _____ .

▶ **Select correct option:**

○ **real**

○ complex

○ zero

○ positive

Below are all the finite difference methods EXCEPT _____.

▶ **Select correct option:**

○ jacobi's method

○ newton's backward difference method

○ **Stirlling formula**

○ Forward difference method

Quiz#10

The central difference method is finite method.

Select correct option:

True

False

Power method is applicable if the eigen vectors corresponding to eigen values are linearly _____.

▶ **Select correct option:**

○ **independent**

○ dependent

Central difference method seems to be giving a better approximation, however it requires more computations.

▶ **Select correct option:**

○ **TRUE**

○ FALSE

The Jacobi iteration _____, if A is strictly diagonally dominant.

▶ **Select correct option:**

○ **converges**

○ diverges

Power method is applicable if the eigen values are _____.

▶ **Select correct option:**

○ **real and distinct**

○ real and equal

○ positive and distinct

○ negative and distinct

determinant of a diagonal matrix is the product of the diagonal elements.

▶ **Select correct option:**

○ **TRUE**

○ FALSE

The Gauss-Seidel method is applicable to strictly diagonally dominant or symmetric positive definite matrices A.

▶ **Select correct option:**

○ **TRUE**

○ FALSE

If n x n matrices A and B are similar, then they have the _____ eigenvalues (with the same multiplicities).

▶ **Select correct option:**

○ **same**

○ different

An eigenvector V is said to be normalized if the coordinate of largest magnitude is equal to _____.

▶ **Select correct option:**

○ **unity**

○ zero

===

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Set-09

Exact solution of 2/3 is not exists.

TRUE

FALSE

The Jacobi's method is

a method of solving a matrix equation on a matrix that has _____ zeros along its main diagonal.

no

atleast one

A 3 x 3 identity matrix have three and _____ eigen values.

same

different

Eigenvalues of a symmetric matrix are all _____ .

real

complex

zero

positive

The Jacobi iteration converges, if A is strictly diagonally dominant.

TRUE

FALSE

Below are all the finite difference methods EXCEPT _____.

 jacobi's method

newton's backward difference method

Stirlling formula

Forward difference method

If n x n matrices A and B are similar, then they have the same eigenvalues (with the same multiplicities).

TRUE

FALSE

If A is a nxn triangular matrix (upper triangular, lower triangular) or diagonal matrix , the eigenvalues of A are the diagonal entries of A.

TRUE

FALSE

The characteristics polynomial of a 3x 3

identity matrix is _____, if x is the eigen values of the given 3 x 3 identity matrix. where symbol ^ shows power.

(x-1)^3

(x+1)^3

 x^3-1

 x^3+1

Two matrices with the same characteristic polynomial need not be similar.

TRUE

FALSE

==

Set-10

/wEPDwUKMTY2I

/wEWBgLDz9rUC

MC090400760 : Imran Shahzad

Time Left **67** sec(s)

Quiz Start Time: 05:13 PM

Question # 1 of 10 (Start time: 05:13:27 PM)   **Total Marks:** 1

The determinant of a _____ matrix is the product of the diagonal elements.

Select correct option:

○ diagonal

○ upper triangular

○ lower triangular

○ scalar

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBALlstSEAv

MC090400760 : Imran Shahzad

Time Left **44** sec(s)

Quiz Start Time: 05:13 PM

Question # 2 of 10 (Start time: 05:14:53 PM)   **Total Marks:** 1

The absolute value of a determinant (|detA|) is the product of the absolute values of the eigenvalues of matrix A

Select correct option:

◯   TRUE

◯   FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBAL85ba6D

MC090400760 : Imran Shahzad

Time Left **78** sec(s)

Quiz Start Time: 05:13 PM

Question # 3 of 10 (Start time: 05:16:13 PM)   **Total Marks:** 1

Central difference method seems to be giving a better approximation, however it requires more computations.

Select correct option:

◯   TRUE

◯   FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBALvyM2J(

MC090400760 : Imran Shahzad

Quiz Start Time: 05:13 PM

Question # 4 of 10 (Start time: 05:17:36 PM)   **Total Marks:** 1

The Power method can be used only to find the eigenvalue of A that is largest in absolute value—we call this eigenvalue the dominant eigenvalue of A.

Select correct option:

⦿   TRUE

○   FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgLv7oHdI

MC090400760 : Imran Shahzad

Quiz Start Time: 05:13 PM

Question # 5 of 10 (Start time: 05:18:12 PM)   **Total Marks:** 1

Let A be an n ×n matrix. The number x is an eigenvalue of A if there exists a non-zero vector v such that _____.

Select correct option:

○   Av = xv

○   Ax = xv

   ○   Av + xv=0

   ○   Av = Ax

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBAKOofDw

MC090400760 : Imran Shahzad

Time Left **40** sec(s)

Quiz Start Time: 05:13 PM

Question # 6 of 10 (Start time: 05:19:22 PM)   **Total Marks:** 1

The Jacobi's method is a method of solving a matrix equation on a matrix that has no zeros along its main diagonal.

Select correct option:

   ○   TRUE

   ○   /wEPDwUKMTY2I

          /wEWBAKyoPefC

MC090400760 : Imran Shahzad

Time Left **78** sec(s)

Quiz Start Time: 05:13 PM

Question # 8 of 10 (Start time: 05:21:26 PM)   **Total Marks:** 1

The Jacobi iteration converges, if A is strictly diagonally dominant.

Select correct option:

◎   TRUE

◎   FALSE

Click here to Save Answer & Move to Next Question

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBgK0iY31C

MC090400760 : Imran Shahzad

Quiz Start Time: 05:13 PM

/wEPDwUKMTY2I

/wEWBAKG4seDI

MC090400760 : Imran Shahzad

Time Left 47 sec(s)

Quiz Start Time: 05:13 PM

Question # 9 of 10 (Start time: 05:22:47 PM)　　　　**Total Marks:** 1

Central Difference method is the finite difference method.

Total Marks: 1

Select correct option:

◎   TRUE

◎   FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2I

/wEWBAKw0/X2E

MC090400760 : Imran Shahzad

Time Left **34** sec(s)

Quiz Start Time: 05:13 PM

Question # 10 of 10 (Start time: 05:23:47 PM)   **Total Marks:** 1

By using determinants, we can easily check that the solution of the given system of linear equation exits and it is unique.

Select correct option:

O   TRUE

O   FALSE

[Click here to Save Answer & Move to Next Question]

(Start time: 05:20:47 PM)

The characteristics polynomial of a 3x 3 identity matrix is _____, if x is the eigen values of the given 3 x 3 identity matrix. where symbol ^ shows power.

Select correct option:

O   (x-1)^3

O   (x+1)^3

O   x^3-1

O   x^3+1

[Click here to Save Answer & Move to Next Question]

==

Set-11

Contents:

Contents:

/w EPDw UKMTY2

/w EWBgKPm46n

Question # 1 of 10 (Start time: 04:40:08 PM)   Total Marks: 1

The determinant of a diagonal matrix is the product of the diagonal elements.
1. **True**
2. False

Question # 2 of 10 (Start time: 04:40:58 PM)   Total Marks: 1

Power method is applicable if the eigen vectors corresponding to eigen values are linearly independent.
1. **True**
2. false

A 3 x 3 identity matrix have three and different eigen values.
1. True
2. **False**

If n x n matrices A and B are similar, then they have the different eigenvalues (with the same multiplicities).
1. True

2. False

The Jacobi's method is a method of solving a matrix equation on a matrix that has _____ zeros along its main diagonal.

1. No
2. At least one

An eigenvector V is said to be normalized if the coordinate of largest magnitude is equal to _____.

1. Unity
2. zero

/w EPDw UKMTY2

The Gauss-Seidel method is applicable to strictly diagonally dominant or symmetric positive definite matrices A.

1. True
2. False

/w EWBgKin4OcE

/w EPDw UKMTY2

The determinant of a _____ matrix is the product of the diagonal elements.

1. Diagonal
2. Upper triangular
3. Lower triangular
4. Scalar

Waisay main nay is ka answer Diagnol keea tha….par yeh charon options theek hain….
You can confirm it from internet…

Jab main yeh MCQ kar raha tha tou tab hi mujhay is par shak ho raha tha….kyun k upper aur lower triangular matrices tou linear algebra mein bhi bahut ziada bataye gaye tou yeh property wahan se hi yaad thi…

Eigenvalues of a symmetric matrix are all _____.

1. Real
2. Zero
3. Positive
4. Negative /w EPDw UKMTY2

/w EWBgKR8tbA

The Power method can be used only to find the eigen value of A that is largest in absolute value—we call this eigen value the dominant eigen value of A.

1. True
2. False

/w EPDw UKMTY2

/w EWBgKjyL19A

The characteristics polynomial of a 3x 3 identity matrix is _____, if x is the eigen values of the given 3 x 3 identity matrix. where symbol ^ shows power.

1. **(x-1)^3**
2. (x+1)^3
3. x^3-1
4. x^3+1

For differences methods we require the set of values.

1. **True**

2. False

If n x n matrices A and B are similar, then they have the different eigenvalues (with the same multiplicities).

1. True

2. **False**

If x is an eigen value corresponding to eigen value of V of a matrix A. If a is any constant, then x – a is an eigen value corresponding to eigen vector V is an of the matrix A - a I.

1. **True**

2. False

a ko agar aap lambda se replace kar dain tou baat clear ho jaye gi....labda ki jagah a use keea gaya hai tou is liay yeh working thora sa confuse karti hai…

Central difference method seems to be giving a better approximation, however it requires more computations.

1. True

2. **False**

Iterative algorithms can be more rapid than direct methods.

1. **True (main nay true hi keea tha, aap isay dekh lena)**

2. False

Central Difference method is the finite difference method.
1. **True**
2. False

/w EPDw UKMTY2

MTH603 Numerical Analysis

Mid Term Examination - Spring 2006
Time Allowed: 90 Minutes

Question No. 1

Mar

ks : 10

Use bisection method to find the solution for
(Perform only three iterations.)

Question No. 2

$$2 + \cos(e^x - 2) - e^x = 0$$

on interval [0.5, 1.5]

Marks : 2

Bisection and false position methods are also known as bracketing method and are always

❺ Divergent
❺ Convergent

Question No. 3

Mar

ks : 10

Use Gauss Elimination method to solve the following system.
$$4x_1 - x_2 + x_3 = 8$$

$$2x_1 + 5x_2 + 2x_3 = 3$$
$$x_1 + 2x_2 + 4x_3 = 11$$

Question No. 4

Mar

ks : 2

The Inverse of a matrix can only be found if the matrix is

❺ Singular
❺ Non singular
❺ Scalar
❺ Diagonal

Question No. 5

Mar

ks : 2

If f (x) contains trigonometric, exponential or logarithmic functions then
this equation is known as

❺ Transcendental equation
❺ Algebraic
❺ Polynomial
❺ Linear

Question No. 6

Mar

ks : 2

In interpolation   is used to represent the δ

❺ Forward difference
❺ Central difference
❺ Backward difference

Question No. 7

Mar

ks : 2

The base of the decimal system is _____

❺ 10
❺ 0
❺ 2
❺ 8
❺ None of the above.

Question No. 8

Use Newton's Raphson Method to find the solution for (Perform only three iterations.)

Question No. 9

Marks : 10

$x^3 + 3x^2 - 1 = 0$ on [-3,-2].

Marks : 10

Approximate f(0.05) by using any of the interpolation technique.

x	0.0	0.2	0.4	0.6	0.8
F(x)	1.000	1.22140	1.49182	1.82212	2.22554

$$[(f_0 + f_n) + 2\sum_{i=1}^{n-1} f_i]$$

is known as

▶ Simpson's 1/3 rd Rule

▶ Simpson's 3/8 rule

▶ Trapezoidal rule

Question No: 2   (Marks: 2)   -   Please choose one

 Bisection method is ……………….. method

► Open Method

► Bracketing Method

Question No: 3   (Marks: 2)   -   Please choose one

 Which method is not used to solve problems related to integration?

► Runge-Kutta Method

► Simpson's $1/3^{rd}$ rule

► Trapezoidal rule.

Question No: 7   (Marks: 10)

 Interpolate the value of 0.25 using Newton's forward difference formula.

x	0.2	0.3	0.4	0.5	0.6
F(x)	0.2304	0.2788	0.3222	0.3617	0.3979

(Perform all the necessary calculation missing calculation and steps may deduct marks.)

Exact solution of 2/3 is not exists.
TRUE
FALSE

The Jacobi's method is
a method of solving a matrix equation on a matrix that has _____ zeros along its main diagonal.

no
atleast one

A 3 x 3 identity matrix have three and _____eigen values.

same
different

Eigenvalues of a symmetric matrix are all _____ .

real
complex
zero
positive

The Jacobi iteration converges, if A is strictly diagonally dominant.

TRUE
FALSE

Below are all the finite difference methods EXCEPT _____.

jacobi's method
newton's backward difference method
Stirlling formula
Forward difference method

If n x n matrices A and B are similar, then they have the same eigenvalues (with the same multiplicities).

TRUE
FALSE

If A is a nxn triangular matrix (upper triangular, lower triangular) or diagonal matrix , the eigenvalues of A are the diagonal entries of A.

TRUE
FALSE

The characteristics polynomial of a 3x 3
identity matrix is _____, if x is the eigen values of the given 3 x 3 identity matrix. where symbol ^
shows power.

(x-1)^3
(x+1)^3
x^3-1
x^3+1

Two matrices with the same characteristic polynomial need not be similar.

TRUE
FALSE

i have attempt mth 603 today........... my paper was power method 5 marks , from vactor 3 marks, Newton_Repshon's forwrd distnce formula 5 marks.,define extra polation 2 marks ,, and most of MCQ's from , Newton_Repshon's method ,or 18 to 22 lec's

question 29: Distinguish between Related and unrelated diversification with example 5marks
question 30 Five porters Model 5 marks
question 31: conecntric diversification two example 3marks
question 32 significant of R&D for an organization 3marks

Which of the following period strategic management was considered to be cure for all problems?
Mid 1950s to mid 1960s
Mid 1960s to mid 1970s
Mid 1970s to mid 1980s
Mid 1980s to mid 1990s

Which of the following is not a pitfall an organization should avoid in strategic planning?
Select correct option:
Failing to involve key employees in all phases of planning
Involving all managers rather than delegating planning to a planner
Top managers not actively supporting the strategic planning process
Doing strategic planning only to satisfy accreditation or regulatory requirements

which of the following are the factors that concern the nature and direction of the economy in which a firm operates?
Select correct option:
Technological
Ecological
Social
Economic

Which of the following best describes this statement; "a Systematic and ethical process for gathering and analyzing information about the competition's activities and general business trends to further a business' own goals"?
Select correct option:
External assessment

Industry analysis
Competitive intelligence program
Business ethics

According to Porter, which strategy offers products or services to a small range of
customers at the lowest price available on the market?
Select correct option:
Low cost
Best value
Cost focus
Differentiation

Long-term objectives includes all of the following <u>EXCEPT:</u>

Measurable

Reasonable

Varying

Consistent

Which one of the following is <u>NOT</u> is a basic mission of a competitive intelligence program?

To provide a general understanding of an industry

To provide a general understanding of a company's competitors

To identify industry executives who could be hired by the firm

To identify potential moves a competitor might make that would endanger a firm

While preparing an External Factor Evaluation Matrix, a total score of 0.8 indicates that:

Firm is taking advantages of strengths and avoiding threats

Firm is taking no advantage of opportunities and is avoiding threats

Firm is not taking advantages of opportunities and is not avoiding threats

Firm is taking advantage of opportunities and is avoiding the threats

Top

Use the Regula-Falsi method to compute a real root of the equation

$$x^3 - 9x + 1 = 0, \quad \text{(Perform two iterations for each part)}$$

(i)   if the root lies between 2 and 4

(ii) if the root lies between 2 and 3.

Solve the following system of equations by Crout's method

$$x_1 + 2x_2 + x_3 = 2$$
$$2x_1 + x_2 - 10x_3 = 0.182$$
$$2x_1 + 3x_2 - x_3 = 2$$

Find the numerical solution of

$$55x - 25z = -200$$
$$-37y - 4z = -250$$
$$-25x - 4y + 29z = 100$$

Using Gauss-Siedel Mehtod with $(1,1,1)|$ as the initial value. Perform two iterations only.

Solve $x^3 - 9x + 1 = 0$ for the root between $x = 2$ and $x = 4$ by the bisection method.
(Perform four iterations only)

Top

Question # 1 of 8 (Start time: 08:34:31 PM)   **Total Marks:** 1

Adams – Bashforth is a multistep method.

🚩 **Select correct option:**

○   TRUE

○   FALSE

> Click here to Save Answer & Move to Next Question

• **Total Marks:** 1

Question # 2 of 8 (Start time: 08:35:01 PM)

Generally, Adams methods are superior if output at _____ points is needed.

🚩 **Select correct option:**

○   many

○ two

○ single

○ at most

▶ Click here to Save Answer & Move to Next Question

Question # 3 of 8 (**Start time: 08:36:02 PM**)   **Total Marks:** 1

In Trapezoidal rule, the integral is computed on each of the sub-intervals by using linear interpolating formula, i.e. for n = 1 and then summing them up to obtain the desired integral.

▶ **Select correct option:**

○ TRUE

○ FALSE

Total Marks: 1

▶ Click here to Save Answer & Move to Next Question

Question # 4 of 8 (**Start time: 08:36:52 PM**)

Euler's Method numerically computes the approximate _____ of a function.

▶ **Select correct option:**

○ antiderivative

○ derivative

○ value

○ error

Click here to Save Answer & Move to Next Question

Total Marks: 1

Question # 5 of 8 (Start time: 08:37:25 PM)

Multistep method does not improves the accuracy of the answer at each step.

► **Select correct option:**

○ FALSE

○ TRUE

Click here to Save Answer & Move to Next Question

Total Marks: 1

Question # 6 of 8 (Start time: 08:37:56 PM)

The trapezoidal rule is a numerical method that approximates the value of a _____.

► **Select correct option:**

○ indefinite integral

○ definite integral

○ improper integral

○ function

▶ Click here to Save Answer & Move to Next Question

Total Marks: 1

Question # 7 of 8 (Start time: 08:38:47 PM)

Simpson's rule is a numerical method that approximates the value of a definite integral by using _____ polynomials.

▶ **Select correct option:**

○ quadratic

○ linear

○ cubic

○ quartic

▶ Click here to Save Answer & Move to Next Question

Total Marks: 1

Question # 8 of 8 (Start time: 08:39:23 PM)

In Simpson's Rule, we use parabolas to approximate each part of the curve. This proves to be very efficient as compared to Trapezoidal rule.

▶ **Select correct option:**

○ TRUE

○ FALSE

Question # 3 of 8 (Start time: 09:46:11 PM)   Total Marks: 1

The first langrange polynomial with equally spaced nodes produced the formula for _____.

▶ Select correct option:

○ Simpson's rule

○ Trapezoidal rule

○ New ton's method

○ Richardson's method

▶ Click here to Save Answ er & Move to Next Question

/w EPDw UKMTY2   /w EWBgLgsdHjB

MC090406505 : Sumera Naz   Time Left   12 sec(s ☾)

Quiz Start Time: 09:44 PM

Question # 4 of 8 (Start time: 09:46:48 PM)   Total Marks: 1

The need of numerical integration arises for evaluating the indefinite integral of a function that has no explicit antiderivative or whose antiderivative is not easy to obtain.

▶ Select correct option:

○   TRUE

○   FALSE

▶   Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2 | /w EWCgKl9pWZI

MC090406505 : Sumera Naz

Time Left 73 sec(s)

Quiz Start Time: 09:44 PM

Question # 5 of 8 (Start time: 09:48:13 PM)   Total Marks: 1

The Trapezoidal Rule is an improvement over using rectangles because we have much less "missing" from our calculations. We used _____ to model the curve in trapezoidal Rule.

▶ Select correct option:

○   straight lines

○   curves

○   parabolas

○   constant

▶   Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2 | /w EWCgLo7tKA A

MC090406505 : Sumera Naz

Time Left   69 sec(s)

Quiz Start Time: 09:44 PM

Question # 6 of 8 (Start time: 09:48:35 PM)   Total Marks: 1

The Euler method is numerically unstable because of _____ convergence of error.

► Select correct option:

○ slow

○ fast

○ moderate

○ no

Click here to Save Answer & Move to Next Question

/w EPDw UKMTY2   /w EWBgLyjl72BA

MC090406505 : Sumera Naz

Time Left   74 sec(s)

Quiz Start Time: 09:44 PM

Question # 8 of 8 (Start time: 09:49:41 PM)   Total Marks: 1

Adams – Bashforth is a multistep method.

► Select correct option:

○ TRUE

○ FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2N   /wEWBgK+3MqIA

Question # 1 of 8 (**Start time: 01:24:49 PM**)   **Total Marks:** 1

The need of numerical integration arises for evaluating the definite integral of a function that has no explicit _____ or whose antiderivative is not easy to obtain.

Select correct option:

◉ antiderivative

○ derivatives

/wEPDwUKMTY2N   /wEWBgLy4ZCpA

Question # 2 of 8 (**Start time: 01:26:18 PM**)   **Total Marks:** 1

In Runge – Kutta Method, we do not need to calculate higher order derivatives and find greater accuracy.

Select correct option:

○ TRUE

◉ FALSE

/wEPDwUKMTY2N   /wEWBgLlwO3cC

Question # 3 of 8 (**Start time: 01:27:35 PM**)   **Total Marks:** 1

An indefinite integral may _____ in the sense that the limit defining it may not exist.

Select correct option:

○ diverge

○ converge

/wEPDwUKMTY2N   /wEWCgKMu86bl

Question # 5 of 8 (**Start time: 01:30:21 PM**)   **Total Marks:** 1

The Trapezoidal Rule is an improvement over using rectangles because we have much less "missing" from our calculations. We used _____ to model the curve in trapezoidal Rule.

▶ **Select correct option:**

◉ straight lines

○ curves

○ parabolas

○ constant

/wEPDwUKMTY2N   /wEWBgK6oIaxBv

Question # 6 of 8 (**Start time: 01:31:18 PM**)   **Total Marks:** 1

An improper integral is the limit of a definite integral as an endpoint of the interval of integration approaches either a specified real number or ∞ or -∞ or, in some cases, as both endpoints approach limits.

▶ **Select correct option:**

◉ TRUE

○ FALSE

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2N

/wEWBgKPg8yyD

Question # 7 of 8 (**Start time: 01:32:33 PM**)   **Total Marks:** 1

Euler's Method numerically computes the approximate derivative of a function.

▶ **Select correct option:**

◉ FALSE

○ TRUE

/wEPDwUKMTY2N   /wEWCgKK6eLRD

Question # 8 of 8 (**Start time: 01:33:57 PM**)   **Total Marks:** 1

If we wanted to find the value of a definite integral with an infinite limit, we can instead replace the infinite limit with a variable, and then take the limit as this variable goes to _____.

▶ **Select correct option:**

○ constant

○ finite

◉ infinity

○ zero

Top

/wEPDwUKMTY2N   /wEWBgK+3MqIA

Question # 1 of 8 (**Start time:**
01:24:49 PM)   **Total Marks:** 1

The need of numerical integration arises for evaluating the definite
integral of a function that has no explicit _____ or whose
antiderivative is not easy to obtain.

▶ **Select correct option:**

○   antiderivative

○   derivatives

▶   Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2N   /wEWBgLy4ZCpA

Time
Left   **59**
             sec(s
)

Question # 2 of 8 (**Start time: 01:26:18 PM**)   **Total Marks:** 1

In Runge – Kutta Method, we do not need to calculate higher order derivatives and find greater accuracy.

▶ **Select correct option:**

○   TRUE

○   FALSE

▶   Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2N   /wEWBgLlwO3cC

Time
Left   **79**
             sec(s
)

Question # 3 of 8 (**Start time: 01:27:35 PM**)   **Total Marks:** 1

An indefinite integral may _____ in the sense that the limit defining it may not exist.

▶ **Select correct option:**

○ diverge

○ converge

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2N   /wEWCgKMu86bI

Time Left **85** sec(s)

Question # 5 of 8 (**Start time: 01:30:21 PM**)   **Total Marks:** 1

The Trapezoidal Rule is an improvement over using rectangles because we have much less "missing" from our calculations. We used _____ to model the curve in trapezoidal Rule.

▶ **Select correct option:**

○ straight lines

○ curves

○ parabolas

○ constant

Click here to Save Answer & Move to Next Question

/wEPDwUKMTY2N   /wEWBgK6oIaxBv

Time Left **83** sec(s)

Question # 6 of 8 (**Start time: 01:31:18 PM**)   **Total Marks:** 1

An improper integral is the limit of a definite integral as an endpoint of the interval of integration approaches either a specified real number or 8 or -8 or, in some cases, as both endpoints approach limits.

▶ **Select correct option:**

○ | TRUE |

○ | FALSE |

▶ Click here to Save Answer & Move to Next Question

| /wEPDwUKMTY2N | /wEWBgKPg8yyD |

Time Left **84** sec(s)

Question # 7 of 8 (**Start time: 01:32:33 PM**)   **Total Marks:** 1

Euler's Method numerically computes the approximate derivative of a function.

▶ **Select correct option:**

○ | FALSE |

○ | TRUE |

▶ Click here to Save Answer & Move to Next Question

| /wEPDwUKMTY2N | /wEWCgKK6eLRD |

Time Left **84** sec(s)

Question # 8 of 8 (**Start time: 01:33:57 PM**)   **Total Marks:** 1

If we wanted to find the value of a definite integral with an infinite limit, we can instead replace the infinite limit with a variable, and then take the limit as this variable goes to _____.

▶ **Select correct option:**

○ | constant |

○ | finite |

○   infinity

○   zero

► | Click here to Save Answer & Move to Next Question |

Top

Top

MTH603-Numerical Analysis
SEMESTER FALL 2005

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Approximate the integral $\int_{1}^{1.6} \frac{2x}{x^2+1} dx$ using Simpson's 1/3 rule and calculate the percentage error. (Take result up to 4 decimal places)

Note: In order to get full marks do all necessary steps.

Construct a forward difference table for the following values

x	0.1	0.3	0.5	0.7	0.9	1.1	1.3
y	0.003	0.067	0.148	0.248	0.37	0.518	0.697

Note : In order to get full marks do all necessary steps.

Solve the system

$4x + 3y = 24$

$3x + 4y - z = 30$

$-y + 4z = -24$

by Gauss Seidal Method, taking $(0, 0, 0)^t$ as initial approximation(Two iterations only and take result up to 4 decimal places)

Note : In order to get full marks do all necessary steps

Let $f(x) = x + \dfrac{2}{x}$, use cubic Lagrange interpolation based on the nodes $x_0 = 0.5, x_1 = 1, x_2 = 2 \ and \ x_3 = 2.5$ to approximate *f(1.5)* and *f(1.3)*.

Note : In order to get full marks do all necessary stepsSolution

Approximate the Dominant Eigenvalue and corresponding Eigenvector for the matrix

$$\begin{bmatrix} 0 & 11 & -5 \\ -2 & 17 & -7 \\ -4 & 26 & -10 \end{bmatrix}$$

by using Power Method. Start with $X_0 = (1,1,1)^t$. (Five iterations only and take result up to 4 decimal places)

Note : In order to get full marks do all necessary steps

Top

NUMERICAL ANALYSIS
Paper Final Term (Held: 25th Feb 2010)

No.	Questions	Mks
1	Find value of given data by Adam Moultan's method	10
2	Find value of given data by Dividend Difference Composite method	10
3	Draw backward difference tables for given Data	5+5
	x				
	y				
	x				
	y				
4	Write Simpson's 1/3 formula	2
5	Find value by Euler's Method	3
6	Find value of K1 by 2nd Order R-K method	2
7	Convergence is used when ----------	1
8	Bisection method is ……………….. method • Bracketing Method • Open • Random	1

	• none http://vustudents.ning.com/	
9	Newton Raphson method is ……………….. method • Bracketing Method • Open • Random • none	1
10	Eigenvalue is • Real • Vector • odd • even	1
11	Find value of y'(1) by Euler's Method taking h=1	2
12	Find value of y'(3) from given table. 	x				 	y					2
13	Find value of y'(0.3) by Lagrange's Method 	x				 	y					3
14	For Simpson's 1/3 rule no.of intervals must be • 1 • 3 • 5 • 8	1
15	For Simpson's 1/3 rule valid no.of intervals are • 1 • 3 • 5 • 8	1
16	For Simpson's 3/8 rule no.of intervals must be • 10 • 11 • 12 • 14 http://vustudents.ning.com/	1
17	Find the value of y'(1) from given forward difference table 	x	y	Δy	Δy^2	Δy^3	 						 						 						 							2

Numerical analysis mth603 paper
Numerical Analysis numerical paper 2009

The paper was very easy.
The mcq's were really easy.
Most of the mcq's were from the last 5 lecture.
and from jacobi's method and other's.
then the logical mcq's like
s inverse * s = I
it was repeated twice.
one 5 mark question was from newton's rapson method
one 10 mark question was from lecture 11 page 69 example
best of luck

NUMERICAL ANALYSIS
Paper Final Term (Held: 25th Feb 2010)

No.	Questions	Mks
1	Find value of given data by Adam Moultan's method	10
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4	Write Simpson's 1/3 formula	2
5	Find value by Euler's Method	3
6	Find value of K1 by 2nd Order R-K method	2
7	Convergence is used when ----------	1
8	Bisection method is ……………….. method • Bracketing Method • Open • Random • none http://vustudents.ning.com/	1
9	Newton Raphson method is ……………….. method • Bracketing Method • Open • Random • none	1
10	Eigenvalue is • Real	1

	• Vector • odd • even			
11	Find value of y'(1) by Euler's Method taking h=1			2
12	Find value of y'(3) from given table. | x			
---	---	---	---
y							2
13	Find value of y'(0.3) by Lagrange's Method | x			
---	---	---	---
y							3
14	For Simpson's 1/3 rule no.of intervals must be • 1 • 3 • 5 • 8			1
15	For Simpson's 1/3 rule valid no.of intervals are • 1 • 3 • 5 • 8			1
16	For Simpson's 3/8 rule no.of intervals must be • 10 • 11 • 12 • 14 http://vustudents.ning.com/			1
17	Find the value of y'(1) from given forward difference table 	x	y	Δy	Δy^2	Δy^3
							2

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best of luck

FINALTERM EXAMINATION
Spring 2010
MTH603- Numerical Analysis (Session - 2)

Ref No: 1508683
Time: 90 min
Marks: 60

Question No: 1   (Marks: 1)   - Please choose one

Symbol used for forward differences is

► ∇

► Δ

► δ

► μ

Question No: 2   (Marks: 1)   - Please choose one

The relationship between central difference operator and the shift operator is given by

► $\delta = E - E^{-1}$

► $\delta = E + E^{-1}$

► $\delta = E^{\frac{1}{2}} + E^{-\frac{1}{2}}$

► $\delta = E^{\frac{1}{2}} - E^{-\frac{1}{2}}$

Question No: 3   (Marks: 1)   - Please choose one

Muller's method requires --------starting points

► 1
► 2
► 3
► 4

Question No: 4   (Marks: 1)   - Please choose one

If   S is an identity   matrix, then

► $S^{-1} = S$

► $S^{t} = S$

► *All are true*

► $S^{-1} = S^{t}$

Question No: 5   (Marks: 1)   - Please choose one

If we retain r+1 terms in Newton's forward difference formula, we obtain a polynomial of degree ---- agreeing with

y_x at $x_0, x_1, \ldots, x_r$

► r+2
► r+1
► r
► r-1

Question No: 6   (Marks: 1)   - Please choose one

———————————————————————————————— P in Newton's forward difference formula is defined as

► $p = (\dfrac{x - x_0}{h})$

► $p = (\dfrac{x + x_0}{h})$

► $p = (\dfrac{x + x_n}{h})$

► $p = (\dfrac{x - x_n}{h})$

Question No: 7   (Marks: 1)   - Please choose one

———————————————————————————————— Octal number system has the base ---------------

► 2
► 8
► 10
► 16

Question No: 8   (Marks: 1)   - Please choose one

———————————————————————————————— Newton's divided difference interpolation formula is used when the values of the independent variable are

► Equally spaced

► Not equally spaced

► Constant
► None of the above

Question No: 9   (Marks: 1)   - Please choose one

———————————————————————————————— Given the following data

x	0	1	2	4
$f(x)$	1	1	2	5

Value of $f(2,4)$ is

    ► 1.5

    ► 3
    ► 2

    ► 1

Question No: 10   (Marks: 1)   - Please choose one

_____ If $y(x)$ is

approximated by a polynomial $p_n(x)$ of degree n then the error is given by

    ► $\varepsilon(x) = y(x) + P_n(x)$

    ► $\varepsilon(x) = y(x) - P_n(x)$

    ► $\varepsilon(x) = P_n(x) - y(x)$

    ► $\varepsilon(x) = y(x) \times P_n(x)$

Question No: 11   (Marks: 1)   - Please choose one

_____ Let I denotes
the closed interval spanned by $x_0, x_1, x_2, x_3, x_4, x_5, x_6, x_7, \overline{x}$. Then $F(x)$ vanishes ------times in the
interval I .

    ► n-1
    ► n+2
    ► n
    ► n+1

Question No: 12   (Marks: 1)   - Please choose one

_____ Differential
operator in terms of forward difference operator is given by

    ► $D = \frac{1}{h}(\Delta + \frac{\Delta^2}{2!} + \frac{\Delta^3}{3!} + \frac{\Delta^4}{4!} + \frac{\Delta^5}{5!} + ...)$

    ► $D = \frac{1}{h}(\Delta + \frac{\Delta^2}{2} + \frac{\Delta^3}{3} + \frac{\Delta^4}{4} + \frac{\Delta^5}{5} + ...)$

    ► $D = \frac{1}{h}(\Delta - \frac{\Delta^2}{2} + \frac{\Delta^3}{3} - \frac{\Delta^4}{4} + \frac{\Delta^5}{5} - ...)$

    ► $D = \frac{1}{h}(\Delta - \frac{\Delta^2}{2!} + \frac{\Delta^3}{3!} - \frac{\Delta^4}{4!} + \frac{\Delta^5}{5!} - ...)$

Question No: 13   (Marks: 1)   - Please choose one

_____ Finding the

first derivative of $f(x)$ at x =0.4 from the following table:

x	0.1	0.2	0.3	0.4
$f(x)$	1.10517	1.22140	1.34986	1.49182

Differential operator in terms of ----------------will be used.

- ► Forward difference operator
- ► Backward difference operator
- ► Central difference operator
- ► None of the given choices

Question No: 14   (Marks: 1)   - Please choose one

For the given table of values

x	0.1	0.2	0.3	0.4	0.5	0.6
$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

$f^{/}(0.1)$, using two-point equation will be calculated as.............

- ► -0.5
- ► 0.5
- ► 0.75
- ► -0.75

Question No: 15   (Marks: 1)   - Please choose one

In Simpson's
1/3 rule, $f(x)$ is of the form

- ► $ax+b$
- ► ax^2+bx+c
- ► ax^3+bx^2+cx+d
- ► $ax^4+bx^3+cx^2+dx+e$

Question No: 16   (Marks: 1)   - Please choose one

While
integrating $I=\int_{a}^{b} f(x)dx$, h , width of the interval, is found by the formula-----.

► $\dfrac{b-a}{n}$

► $\dfrac{b+a}{n}$

► $\dfrac{a-b}{n}$

► None of the given choices

Question No: 17   (Marks: 1)   - Please choose one

To apply Simpson's 1/3 rule, valid number of intervals are.....

► 7
► 8
► 5
► 3

Question No: 18   (Marks: 1)   - Please choose one

For the given table of values

x	02	0.3	0.4	0.5	0.6	0.7
$f(x)$	0.425	0.475	0.400	0.452	0.525	0.575

$f''(0.2)$, using three-point equation will be calculated as ……

► 17.5
► 12.5
► 7.5
► -12.5

Question No: 19   (Marks: 1)   - Please choose one

To apply Simpson's 1/3 rule, the number of intervals in the following must be

► 2
► 3
► 5
► 7

Question No: 20   (Marks: 1)   - Please choose one

To apply Simpson's 3/8 rule, the number of intervals in the following must be

► 10
► 11
► 12
► 13

Question No: 21   (Marks: 1)   - Please choose one

If the root of the given equation lies between a and b, then the first approximation to the root of the equation by bisection method is ……

► $\dfrac{(a+b)}{2}$

► $\dfrac{(a-b)}{2}$

► $\dfrac{(b-a)}{2}$

► None of the given choices

Question No: 22   (Marks: 1)   - Please choose one

...........lies in the category of iterative method.

► Bisection Method
► Regula Falsi Method
► Secant Method
► None of the given choices

Question No: 23   (Marks: 1)   - Please choose one

For the equation $x^3 + 3x - 1 = 0$, the root of the equation lies in the interval......

► (1, 3)
► (1, 2)
► (0, 1)
► (1, 2)

Question No: 24   (Marks: 1)   - Please choose one

Rate of change of any quantity with respect to another can be modeled by

► An ordinary differential equation
► A partial differential equation

► A polynomial equation

► None of the given choices

Question No: 25   (Marks: 1)   - Please choose one

If
$$\frac{dy}{dx} = f(x, y)$$

Then the integral of this equation is a curve in

► None of the given choices

► xt-plane
► yt-plane
► xy-plane

Question No: 26   (Marks: 1)   - Please choose one

In solving the differential equation
$$y^{/} = x + y \ ; \ y(0.1) = 1.1$$
$h = 0.1$, By Euler's method $y(0.2)$ is calculated as

► 1.44
► 1.11
► 1.22
► 1.33

Question No: 27   (Marks: 1)   - Please choose one

In second order Runge-Kutta method
k_1 is given by

► $k_1 = hf(x_n, y_n)$

► $k_1 = 2hf(x_n, y_n)$

► $k_1 = 3hf(x_n, y_n)$

► None of the given choices

Question No: 28   (Marks: 1)   - Please choose one

In fourth order Runge-Kutta method, k_2 is given by

► $k_2 = hf(x_n + \dfrac{h}{2}, y_n + \dfrac{k_1}{2})$

► $k_2 = hf(x_n + \dfrac{h}{3}, y_n + \dfrac{k_1}{3})$

► $k_2 = hf(x_n - \dfrac{h}{3}, y_n - \dfrac{k_1}{3})$

► $k_2 = hf(x_n - \dfrac{h}{2}, y_n - \dfrac{k_1}{2})$

Question No: 29   (Marks: 1)   - Please choose one

In fourth order Runge-Kutta method, k_4 is given by

▶ $k_3 = hf(x_n + 2h, y_n + 2k_3)$

▶ $k_3 = hf(x_n - h, y_n - k_3)$

▶ $k_3 = hf(x_n + h, y_n + k_3)$

▶ None of the given choices

Question No: 30   (Marks: 1)   - Please choose one

Adam-Moulton P-C method is derived by employing

▶ Newton's backward difference interpolation formula
▶ Newton's forward difference interpolation formula
▶ Newton's divided difference interpolation formula
▶ None of the given choices

Question No: 31   (Marks: 2)

If $F(h) = 256.2354$ and $F(\frac{h}{2}) = 257.1379$, then find $F_1(\frac{h}{2})$ using Richardson's extrapolation limit.

Question No: 32   (Marks: 2)

Evaluate the integral

$$\int_0^{\frac{\pi}{2}} (\cos x + 2)dx$$

Using Simpson's 3/8 rule

Take h= $\dfrac{\pi}{4}$

Question No: 33   (Marks: 2)

Write a general formula for Modified Euler's method of solving the given differential equation.

Question No: 34   (Marks: 3)

Evaluate the integral

$$\int_0^4 x^2 dx$$

Using Trapezoidal rule
Take h=1

Question No: 35   (Marks: 3)

Evaluate the integral

$$\int_3^5 (\log x + 2)dx$$

Using Simpson's 3/8 rule
Take h=1

Question No: 36   (Marks: 3)

Write a formula for finding the value of k_3 in Fourth-order R-K method.

Question No: 37   (Marks: 5)

Find Newton's forward difference table from the following data.

x	0.0	0.1	0.2	0.3	0.4
$f(x)$	1	0.9048	0.8187	0.7408	0.6703

Question No: 38   (Marks: 5)

Evaluate the integral

$$\int_0^3 (x^2 + x)dx$$

Using Simpson's 3/8 rule

Take h=1

Question No: 39   (Marks: 5)

Use Runge-Kutta Method of order four to find the values of
$k_1, k_2, k_3 \text{ and } k_4$ for the initial value problem

$y^/ = \dfrac{1}{2}(2x^3 + y), y(1) = 2$ taking $h = 0.1$

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